



Proceedings

ASPE 2015 Summer Topical Meeting

Precision Interferometric Metrology

July 8-10, 2015

American Society for

Precision Engineering

P.O. Box 10826, Raleigh, NC 27605-0826
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Proceedings of the
5th ASPE Topical Meeting
on
Precision Interferometric
Metrology

July 8-10, 2015

The Golden Hotel
Golden, Colorado, USA

The American Society for Precision Engineering (ASPE) is a multidisciplinary professional and technical society concerned with research and development, design, manufacture and measurement of high accuracy components and systems. ASPE activities encompass relevant aspects of mechanical, electronic, optical and production engineering, physics, chemistry, and computer and materials science. Membership is open to anyone interested in any aspect of precision engineering.

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Preface

This book comprises the proceedings of the 5th ASPE Topical Meeting on **Precision Interferometric Metrology**. The contributions reflect the authors' opinions and are published as presented to ASPE without change. Their inclusion in this publication does not necessarily constitute endorsement by the ASPE or its editorial staff.

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Foreword

The Fifth ASPE Topical Meeting on **Precision Interferometric Metrology** is being held at The Golden Hotel in Golden, Colorado. The Golden Hotel overlooks the scenic Clear Creek and the Rocky Mountain Foothills and is conveniently located in the heart of the historic downtown Golden, Colorado.

The meeting brings together specialists and practitioners from industry, government and academia in an ideal forum for the exchange of ideas. The emphasis will be on discussion in a relaxed atmosphere. In addition to invited speakers, the program consists of oral presentations in the highly-successful workshop format enjoyed in past meetings. The committee has planned a technical tour at NIST in Boulder prior to the conference on Tuesday, July 7. ASPE has arranged for a discount for many activities through Denver Adventures on Saturday, July 11. The workshop format of the conference will lend itself to many social events such as breakfasts, lunches, a dinner and a cocktail hour.

Meeting Organizers

Christopher J. Evans, University of North Carolina at Charlotte

Peter J. de Groot, Zygo Corporation

Angela D. Davies, University of North Carolina at Charlotte

Jack Clark, Surface Analytics, LLC; Colorado State University

Scientific Committee

Vivek G. Badami, Zygo Corporation

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Program & Schedule

Date	Time Begin	Time End	Event	Location (The Golden Hotel)
Tue Jul 7				
	08:00	02:00	Optional Tour of National Institute of Standards and Technology (Boulder)	Meet in Hotel Lobby for Bus (Bus Leaves at 8:00 AM)
Wed Jul 8				
	08:00	09:00	Breakfast Buffet	Golden Vista Ballroom
	09:00	10:00	Welcome & Technical Session I	Clear Creek Ballroom
	10:00	10:30	Coffee Break	Just Outside of Clear Creek
	10:30	12:00	Technical Session II	Clear Creek Ballroom
	12:00	01:30	Lunch	Golden Vista Ballroom
	01:30	03:00	Technical Session III	Clear Creek Ballroom
	03:00	03:30	Coffee Break	Just Outside of Clear Creek
	03:30	05:00	Technical Session IV	Clear Creek Ballroom
	06:00	08:00	Conference Dinner	Creekside Courtyard
Thu Jul 9				
	08:00	09:00	Breakfast Buffet	Golden Vista Ballroom
	09:00	10:00	Technical Session V	Clear Creek Ballroom
	10:00	10:30	Coffee Break	Just Outside of Clear Creek
	10:30	12:00	Technical Session VI	Clear Creek Ballroom
	12:00	06:30	Free Afternoon	
	06:30	07:30	Cocktail Reception	Golden Vista Ballroom
	07:30	09:00	Evening Session	Golden Vista Ballroom
Fri Jul 10				
	08:00	09:00	Breakfast Buffet	Golden Vista Ballroom
	09:00	10:00	Technical Session VII	Clear Creek Ballroom
	10:00	10:30	Coffee Break	Just Outside of Clear Creek
	10:30	12:00	Technical Session VIII & Discussion, Photos & Closure	Clear Creek Ballroom
	12:00	01:30	Lunch	Golden Vista Ballroom
Sat Jul 11				
			Optional Denver Adventure Tours	Meet Hotel Lobby

National Institute of Standards and Technology Tour

325 Broadway Street • Boulder, CO 80305

Tuesday, July 7, 2015

Cost: \$50 (includes Lunch)

- The Charter Bus will leave The Golden Hotel at 8:00 AM (Meet in Hotel Lobby by 7:55 AM)
- NIST Tour will be from 9:00 AM – 12:00 Noon
- Lunch will be provided
- All participants will need a picture ID. If US resident, Driver's License ID is acceptable. If not a US resident, you will need a passport.

NIST Tour Highlights

Frequency Comb Atmospheric Gas Measurements. Current fielded gas sensors lack the accuracy and widespread regional coverage to support future regulatory actions and the ability to identify localized sources of greenhouse gases. NIST has demonstrated a revolutionary spectroscopy tool that can detect variations in greenhouse gasses across a 2-km path using a fieldable frequency comb. Working closely with NOAA's Carbon Cycle and Greenhouse Gases Group led by Dr. Pieter Tans, researchers demonstrated the NIST tool is ideally suited to the challenges associated with accurate sensing of atmospheric trace gases with broad spectral coverage for accurate multi-species detection. Equally important, its high acquisition speed is insensitive to weather changes at different times and locations. Future collaborative efforts will focus on actuate monitoring of multiple gas species over multiple optical paths, supporting ground validation along satellite spectrometer ground tracks, and round-the-clock monitoring of CO₂ sequestration sites, landfills, cities, or other sites for which infrequent satellite coverage is less desirable. These same NIST techniques can support monitoring of sequestration sites, energy exploration, biosphere interaction studies, and trace gas detection for U.S. Department of Defense applications.

Comb Calibrated LADAR for Ranging and Imaging. Non-contact surface profiling is of general interest in many fields from precision manufacturing to forensics. One well-established technique is laser ranging and detection (LADAR). NIST developed a new scanning imaging frequency modulated continuous wave (FMCW) LADAR whose range measurements are referenced directly to the tooth spacing of a free-running frequency comb (a mode-locked laser). In that way, the measured range is directly traceable to an rf standard through the frequency comb's pulse repetition rate, rather than to a physical artifact such as an interferometer or etalon, whose properties can change with environmental conditions. The system operates under ambient lighting conditions, is eye-safe and offers a high dynamic range in return power, enabling measurements of surfaces with vastly different reflectance with the same precision at a one pixel per 0.5 ms update rate.

Laser Welding Standards. With industrial and academic partners, NIST is using advanced metrology tools to transform laser welding from art to science. Welding is involved in products that represent 50 percent of the U.S. GNP — from cars to buildings to personal electronics as well as implantable medical devices. Laser welding is more cost effective and environmentally friendly than traditional welding methods. Yet, laser welding is under-utilized due to a lack of

understanding of the underlying science necessary for optimization and widespread adoption. Four of the five top manufacturers of lasers for welding are U.S. Companies. Yet, China and Germany lead the U.S. in the use of laser welding for manufacturing. The NIST Laser Welding Program has drawn resources from the technical expertise found at NIST in materials science, physics, chemistry, and mechanical engineering uniting expertise in materials characterization and laser metrology to demonstrate a novel approach to laser welding and in situ monitoring; this combination of expertise is not found anywhere else in the world.

PIF and Nanofabrication Facility. NIST maintains a research-class facility for fabrication of superconductor and nanoelectronic circuits. Beginning with computer-aided design, NIST researchers use electron beam lithography to make structures smaller than 50 nanometers and optical lithography to make complex circuits containing as many as 330,000 Josephson junctions.

The 9,000-square-foot clean room includes tools for fabricating microelectromechanical systems (MEMS), which are essential for many ultra-sensitive instruments and micromachined ion traps for future atomic clocks and some types of quantum computing. NIST also has created unique experimental systems for characterizing devices and developing new measurement instruments. These include refrigerators that can cool down to temperatures of 30-50 milliKelvin.

The PML's imaging facility provides new capabilities for precisely measuring the structure and chemical composition of materials at sub-nanometer scales, and for studying and creating technologically important materials and devices at the atomic level.

The one of a kind collection of imaging apparatus include a helium ion microscope, which provides a new method for imaging surfaces of biological and inorganic materials; a transmission electron microscope, which offers the resolution needed to characterize crystalline materials used for nanoscale and quantum light sources and quantum information circuits; a focused ion beam/scanning electron microscope that can selectively mill devices and materials to reveal their internal structure; and a field ion microscope, or atom probe, capable of revealing the location and chemical composition of structures invisible to other methods.

The predecessor to NIST's nanofab facility was also responsible for building the amplification devices used to detect the earliest echoes of the Big Bang, confirming a decades-old hypothesis that describes the universe's ultrafast expansion during its first moments. The findings provide researchers with the first direct measurement of conditions at nearly the instant that cosmic expansion began, and may have far-reaching implications for physicists' understanding of general relativity, quantum mechanics and the origin of the universe.

Cocktail Hour and Evening Session

Thursday, July 9, 2015

Cocktail Hour: 6:30 PM - 7:30 PM (Golden Vista)

Evening Session: 7:30 PM - 9:00 PM (Golden Vista)

Session Moderators: Christopher J. Evans, University of North Carolina-Charlotte, Vivek G. Badami, Zygo Corporation and Peter J. de Groot, Zygo Corporation

Immediately following the cocktail hour, the tradition of an evening session of free flowing discussion, short reports of latest results, and provocative presentations on needs and technical directions in the field will take place. Please bring a couple of PowerPoint slides and come prepared to participate.

Denver Adventure Tours

Saturday, July 11, 2015

ASPE has arranged a discount through Denver Adventure Tours. Interested parties should contact Denver Adventure Tours directly at (303) 984-6151 to arrange an adventure that you will enjoy and to pay the fee. The discount code is ASPE2015.

Zipline Tour – Ride down Colorado’s longest and fastest ziplines ranging from 850 ft. to over 1,900ft long, with speeds up to 60 mph and up to 250 ft. above ground! Our zipline course is located in a beautiful nature park nestled in the Rocky Mountains only 30 minutes from Denver.

Whitewater Rafting – Experience whitewater rafting and you will surely come back for more! You will raft through a beautiful canyon, past a gold mine, and view the occasional wildlife. Rafting trips at all levels are available from beginner family trips to advanced trips for experienced rafters. We will handle all the logistics and gear. All you need to do is enjoy the ride!

Hiking – The Rocky Mountains west of Denver, commonly referred to as the “Front Range”, are a playground for a variety of outdoor activities. Whether you just want to escape the city for a few hours or for a full day, we can put together a trip that fits your needs. Individuals, small, and large groups are welcome.

Colorado Springs Sightseeing Tour – Details can be obtained at (303) 984-6151.

Information – Courtesy Denver Adventure Tours

Invited Speakers

Vivek G. Badami, Zygo Corporation

TBA

Esther Baumann, National Institute of Standards & Technology

Comb Calibrated FMCW LADAR for Ranging and Surface Mapping

James H. Burge, University of Arizona

Precision Metrology for Large Telescope Mirrors

Jack Clark, Surface Analytics, LLC and Colorado State University

Interferometric Metrology for Precision Engineering – a Report from the Front Lines

Christopher J. Evans, University of North Carolina at Charlotte

A Flexible Interferometric Metrology System for Litho Optics

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Thursday, July 9, 2015, 7:30 PM - 9:00 PM

Session Moderators: Christopher J. Evans, University of North Carolina -Charlotte, Vivek G. Badami, Zygo Corporation and Peter J. de Groot, Zygo Corporation

A tradition at ASPE's Topical Meetings on interferometry is an evening session of free flowing discussion, short reports of latest results, and provocative presentations on needs and technical directions in the field. Bring a couple of PowerPoint slides and come prepared to participate.

Session VII

Friday, July 10, 2015, 9:00 AM - 10:00 AM

Session Chair: Christopher J. Evans, University of North Carolina -Charlotte and Vivek G. Badami, Zygo Corporation

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Session Chair: Jack Clark, Surface Analytics, LLC; Colorado State University

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Technical Papers

DEVELOPMENTS IN PRECISION INTERFEROMETRY

Klaus Freischlad
Conjugate Design and Engineering LLC
Tucson, AZ

INTRODUCTION

Precision interferometry is an important tool for the measurement of surface topography at sub-nm, and even sub-Angstrom, levels. Examples for applications are hard disk substrates and silicon wafers, as well as the surfaces of optical elements for micro-lithography. The measurement of the waviness, or mid-spatial frequency content, of these surfaces is of special importance, where the spatial frequency band of interest depends on the application, but typically ranges from about 0.5mm to 5mm.

In this presentation an overview is given over some lesser known interferometer designs from the last 20 years addressing these measurement requirements. They each require a system design that minimizes the effect of potential error sources, as well as a method to accurately calculate optical phase maps from fringe intensities.

The theory to derive and analyze phase algorithms is well developed, originally leading to algorithms of the linear filtering type [1, 2]. Recently algorithms with more extensive processing have been published which eliminate the effect of vibrations during the measurement [3]. Although very important, they are not the focus of this presentation, but rather the optical design principles of the presented interferometers will be discussed in more detail.

In two-beam interferometers the test wave is always compared the reference wave, and the shape of the reference wave also appears in the measurement result. We can write for the measured topography map:

$$M(x, y) = T(x, y) - R(x, y) + N(x, y) \quad , \quad (1)$$

where $M(x, y)$ is the measured map, $T(x, y)$ is the test surface topography, $R(x, y)$ is the system reference map, and $N(x, y)$ is the random noise contribution. Typically R is a static bias that can be calibrated and subtracted, whereas N is uncorrelated random noise whose effect can be

reduced by averaging. If all three quantities T , R , and N are uncorrelated, their variances σ^2 add and we obtain for the variance of the measurement

$$\sigma_M^2 = \sigma_T^2 + \sigma_R^2 + \sigma_N^2 \quad . \quad (2)$$

For small errors R and N we have for the RMS:

$$\sigma_M \approx \sigma_T \left(1 + \frac{1}{2} \frac{\sigma_R^2 + \sigma_N^2}{\sigma_T^2} \right) = \sigma_T \left(1 + \frac{1}{2} \left(\frac{\sigma_E}{\sigma_T} \right)^2 \right) \quad (3)$$

where the error variance σ_E^2 is the sum of the variances of R and N . If the RMS σ_E of the combined error is less than 30% of the true surface-RMS, the RMS of the measured map represents the true surface RMS with less than 5% error. We would like to control measurement errors at least to this level.

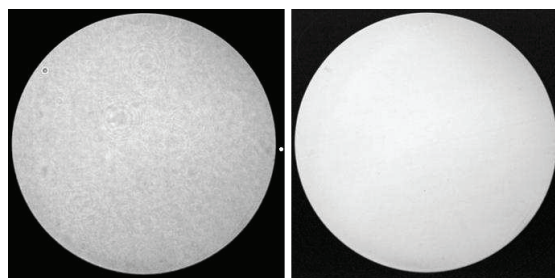


FIGURE 1. *Coherent noise. Left: Coherent illumination; Right: Partially coherent illumination*

Coherent noise in the optical system presents a difficult error case. Depending on the conditions, it is neither just a static bias to be calibrated and subtracted, nor a random error that is easily reduced by averaging. With a laser as coherent light source coherent noise is created in the interferometer by forward scattering at surface imperfections of the interferometer optics. Figure 1 shows on the left an interferometer field illuminated with a spatially coherent laser source, where the grainy appearance and

presence of bull's eye patterns demonstrate the coherent noise, mostly at mid to higher spatial frequencies. On the right is shown the same field with a low-coherence extended-source illumination, resulting in a much smoother appearance.

To demonstrate the effect of the coherent noise, measurements of a 4" flat were taken at various tilts with an interferometer with a coherent source and with an interferometer with an incoherent source. Figure 2 [adapted from 4] shows the RMS in two waviness bands of the difference maps vs tilt, where a map with zero tilt was subtracted from the maps with tilt. We see that for two well-nulled maps the difference map shows an RMS of 0.1nm for both coherent and incoherent illumination, and represents the baseline system noise. However, even for small tilts the RMS rises rapidly for coherent illumination. This first steep rise is attributed to speckle de-correlation between the maps, whereas the slower rise after 1-2 fringes of tilt may stem from retrace error due to non-identical path of the test and reference beam through the interferometer optics. Note that for incoherent illumination the difference RMS stays essentially constant near 0.1nm.

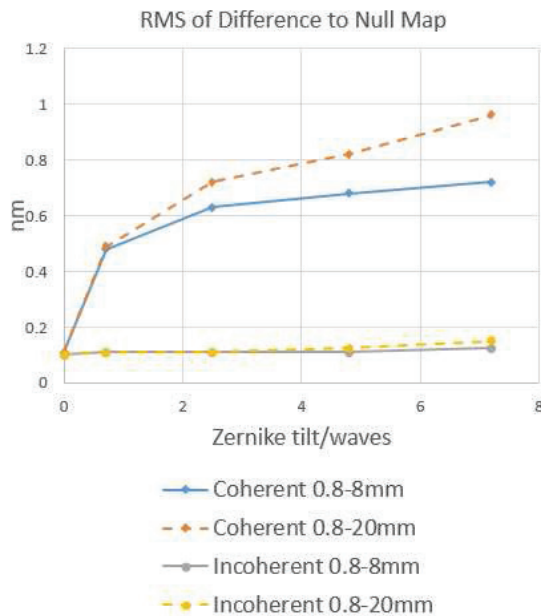


FIGURE 2. Difference map RMS in waviness band for coherent and incoherent illumination.

This strong tilt dependence of the coherent noise makes a calibration and subtraction very difficult. The long-range figure of a test surface

may show several microns departure in addition to the waviness with nm level amplitudes. Thus the local slopes may change across the surface and from test piece to test piece, and a complete elimination of the coherent noise is not possible by calibration.

It was quickly realized after the first lasers were used for optical metrology in the 60s that coherent noise can be a problem. There exist well-known methods of reducing the spatial coherence of laser sources that have been implemented in commercial interferometers, such as using an extended spot (MiniFIZ from Phase Shift Technology) or ring on a rotating ground glass (Verifire from Zygo), or to average phase measurements with different illumination angles (Direct100 from Zeiss [5], DynaFiz from Zygo). Also, a rotating ground glass was used in an intermediate image of the test surface to obtain incoherent imaging at least through the final zoom lens before the fringe camera (MarkII from Zygo).

However, the interferometers presented here all employ an extended white-light source with low spatial and temporal coherence to provide reliable measurements of waviness in the sub-nm, and even sub-Angstrom range, as are usually obtained only by white-light microscopic profilers over much smaller areas.

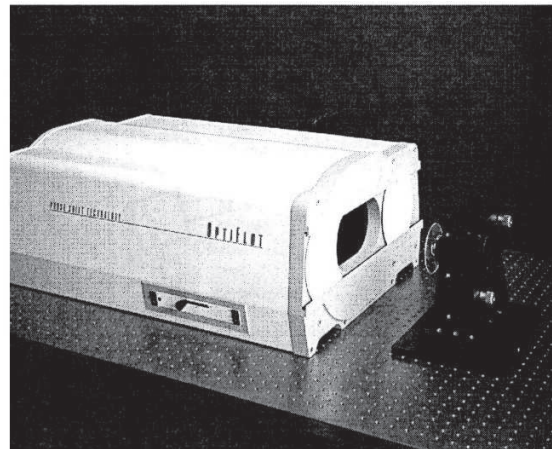


FIGURE 3. OptiFlat with glass disk as test piece.

OPTIFLAT

In the mid-90s the OptiFlat interferometer [6, 7] was developed by Phase Shift Technology for measuring the figure and waviness of thin glass substrates, such as for flat panel displays and glass disks, see figure 3. The interference of the

light reflected at the back side had to be suppressed, and the measurement error at mid-spatial frequencies had to be reliably well below one nm. A phase-shifting Mach-Zehnder interferometer with a white-light, extended source was chosen as interferometer configuration. During its product life OptiFlat was, to the author's knowledge, the only commercial large-aperture white-light interferometer.

The optical schematic of OptiFlat is shown in figure 4. The test diameter is 100mm, with an angle of incidence of the axial ray on the test surface of 15°. The cavity is strictly symmetrical between test and reference arm, and full fringe contrast is achieved for white light. The illumination consists of a halogen lamp which is coupled to the interferometer by a ¼" diameter fiber bundle. The exit face of the fiber bundle is imaged by the large condenser lens onto the aperture stop of the imaging objective, overfilling it by a factor of two. Since the source is spatially incoherent, the quality of the condenser lens is not critical, and a Fresnel lens was used.

The fringes are mainly localized by temporal coherence, and with the interference filter (656nm center wavelength, 3nm bandwidth) the temporal coherence depth is +/- 35 µm around the zero path difference position. Thus glass substrates with a thickness down to 100 µm can be measured without backside influence. A special alignment path complementary to the fringe path allows for easy alignment of tip/tilt and axial position of the test surface.

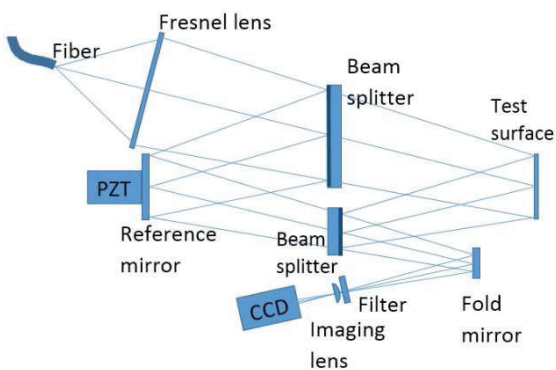


FIGURE 4. Optical schematic of OptiFlat.

The Instrument Transfer Function (ITF), i.e. the transfer function characterizing the transfer of spatial frequencies from surface height to measured surface map, was determined by measuring a sharp step and comparing the

result to the ideal case. The ITF in figure 5 is close to the characteristic shape of ideal incoherent imaging, with more than 50% transfer for periods longer than 0.5mm.

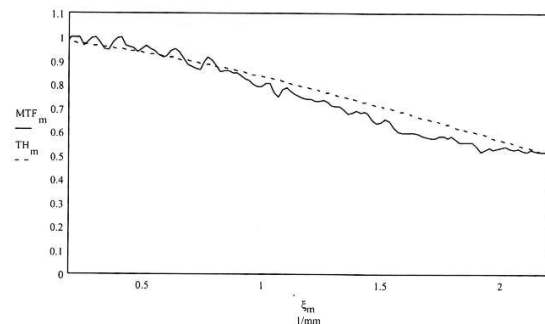


FIGURE 5. OptiFlat Instrument Transfer Function. Solid line: Measured; dashed line: model. ξ is the spatial frequency.

The system noise is determined by looking at the difference between two maps where the test surface topography drops out. Its power spectral density (PSD) divided by 2 represents the PSD of the system noise. Integrating the PSD over the spatial frequency band of interest yields the RMS system noise in this band.

Figure 6 shows the Noise Equivalent Power Spectral Density (NEPSD) for averages of 16 maps. The NEPSD is given by the PSD of the system noise divided by the squared ITF. Thus the NEPSD is equal to the PSD of a test surface producing a map with the same PSD as the measurement noise. For the waviness band from 0.8mm to 20mm periods it is around 0.005nm²/mm, leading to an RMS waviness noise of 0.08nm. The variation of the RMS noise with tilt is described by the partially coherent source case of figure 2, essentially showing no increase with tilt.

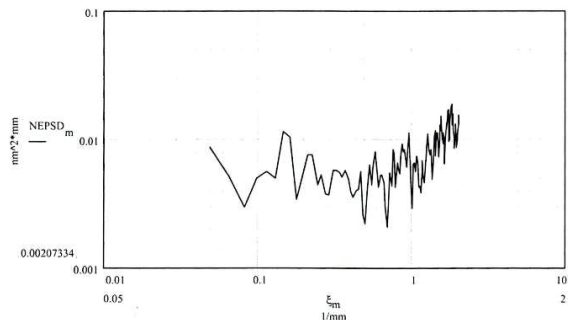


FIGURE 6. Noise Equivalent Power Spectral Density (averages of 16).

Thus OptiFlat has the resolution and low noise required for the waviness measurement. A waviness map of 95mm disk substrate is shown in figure 7, with an RMS of 1.6nm.

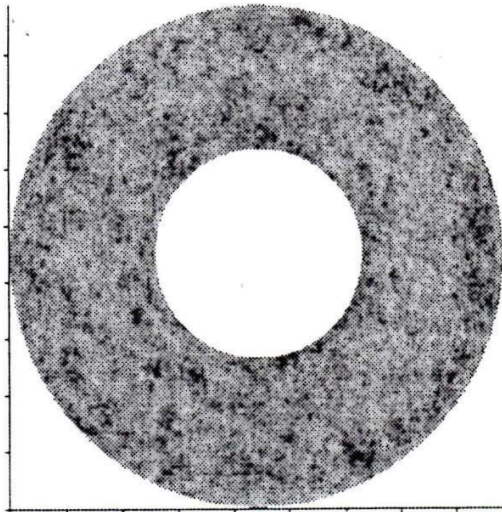


FIGURE 7. Waviness map of a 95mm disk from OptiFlat. RMS = 1.6nm.

ASPHERE STITCHING INTERFEROMETER

An interferometer for the waviness measurement of aspheric surfaces was developed in early 2000 by ADE Phase Shift for a customer working on EUV optics. The surface accuracy requirements of EUV optics are now extreme, with a target RMS of 50 pm for each decade of spatial frequencies [8]. The interferometer, shown in figure 8, consists of a phase-shifting Twyman-Green interferometer where again an extended, filtered white-light source is used to eliminate all coherent noise from the measurements. The measurement field can be set to 10x10mm, 20x20mm, and 30x30mm, and the whole test surface is covered by stitching of sub-apertures.

The optical system schematic is shown in figure 9. The light from a fiber bundle is collimated and passes through an interference filter and beam splitter to the test arm with the diverger and test surface. The beam reflected at the beam splitter is the reference beam. The returning test and reference beams are transferred to the camera by a telecentric relay. An alignment lens can be brought into the beam before the camera to image the source, and thus aid the initial coarse alignment of the interferometer to the test surface.

A diverger has to be used in the test arm to create a spherical wave matching the radius of curvature of the test surface. In order to obtain high-contrast fringes, not only has the group-path of the test arm and reference arm to be matched because of limited temporal coherence, but the source extent requires the axial shift and lateral shear between the two detector images in source space to be close to zero [9]. This is the same requirement as in OptiFlat, but there all cavity optics including the test surface were plane, and the above coherence requirements are easily met by symmetry between test and reference arm.



FIGURE 8. Aspheric Stitching Interferometer with diverger and test surface. The shown mount is a fixed test stand, not the stitching stage.

Across the diameter of an aspheric surface the local radius of curvature can change significantly from the nominal best fitting radius, and we need to be able to adapt to this varying curvature during the acquisition of the sub-apertures for the stitching. In a common Fizeau interferometer with transmission sphere, the axial position of the test surface is adjusted for adapting to the test curvature and for removing the power fringes. This cannot be done here since then the optical group path of the test arm would not

match the reference arm anymore. Instead, the alignment takes place in two straightforward steps: First the test surface position is adjusted with respect to the interferometer to match the group path and obtain high contrast fringes. Then just the diverger is moved axially to null the power fringes. Since the diverger is used in transmission, this does not change the group path of the test arm.

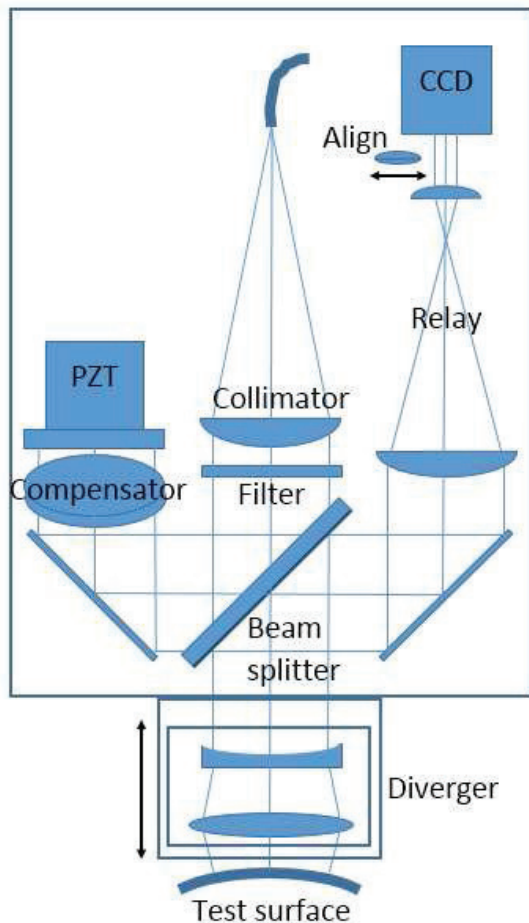


FIGURE 9. Optical schematic of the Asphere Stitching Interferometer, shown with a convex test surface. The diverger is adjusted axially on a translation stage attached to the mainframe.

To make the axial shift of the CCD-image in source space independent of the diverger position, a special design form is required for the divergers, see figure 10. The diverger images the CCD-image created by the relay onto the test surface with 1:1 magnification. Hence the CCD-image and test surface are located in the principal planes of the diverger, and the focal length of the diverger is equal to the nominal test

radius. For the 1:1 magnification also the longitudinal magnification is one, and the image does not move significantly when changing diverger position. For a test radius in the 300mm range, a radius change of up to $\pm 10\%$ can be accommodated with only minor loss of fringe contrast for the f/50 source extent of the interferometer. During the design process of the diverger these first order constraints need to be met, as well as targets for wavefront, imaging quality, and distortion. In addition, the principal plane spacing and the group path need to be matched to the reference arm [10].

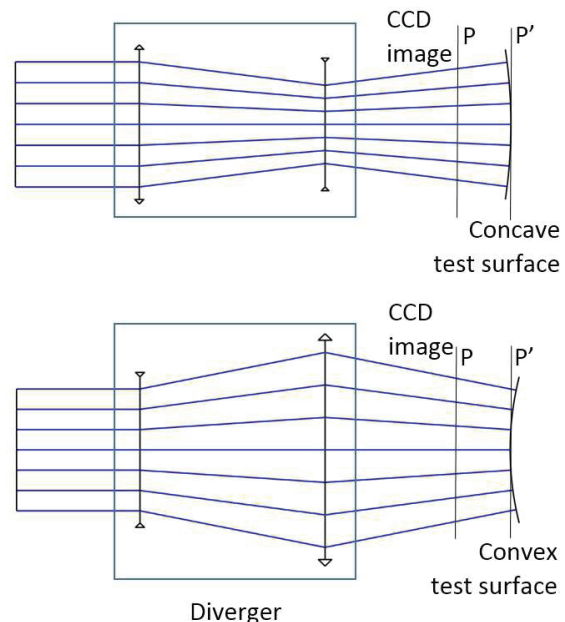


FIGURE 10. Diverger imaging. P , P' are the principal planes.

Nevertheless the designs are fairly simple with 2 to 3 elements. Because of the coherence requirements, the divergers are matched to the test radius, but can handle a significant radius range. For testing flats the diverger contains a plane-parallel plate of the proper thickness. The wavefront error is calibrated using a spherical surface, and the calibration does not change with diverger position. Since the test surface image does not move with diverger position, its image on the CCD is always in focus, and the ITF is maintained for the waviness frequencies independent of test surface radius.

Due to the absence of optical phase noise the Asphere Stitching Interferometer provides very low noise waviness measurements for spatial periods 0.3m to 5mm, similar to the large-

aperture OptiFlat, but for curved surfaces. Figure 11 shows the PSD of the system noise, with an RMS waviness noise of 0.044nm. With the special diverger design the system is straightforward to use, and accommodates the radius variations of aspheric surfaces. Today this system is still being used successfully.

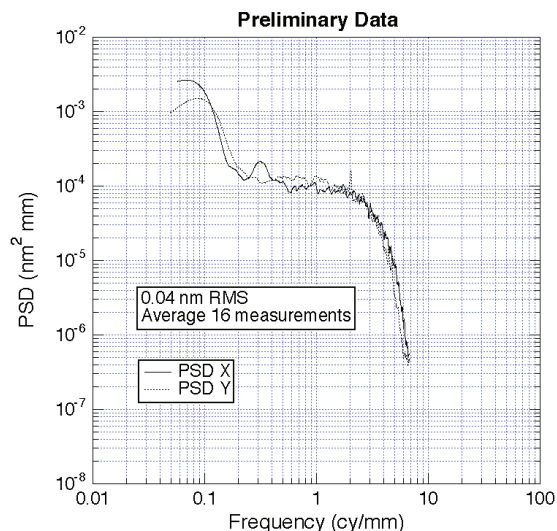


FIGURE 11. PSD of measurement noise for 20x20mm field.

VIRTUAL REFERENCE INTERFEROMETER

So far we were concerned with coherent noise. In the next interferometer we still use an extended white-light source, but also want to remove the contribution of the system reference map to the final measurement result.

When measuring super-polished surfaces with roughness and waviness at the Angstrom level or below, it is not feasible to have a reference surface significantly better than the test surface. While there exist a number of calibration techniques to determine the system reference and subtract it, an interferometer configuration is described here where, at least at mid and high spatial frequencies, the system reference does not contribute to the measurements. It is based on a virtual reference configuration, which was first described over 50 years ago by Schulz [11].

Figure 12 shows the optical schematic of our design [12]. There are 2 beam splitters. The test beam is transmitted by both beam splitters until it is reflected at the test surface, and then reflected by the second beam splitter to the relay and the camera. The reference beam is reflected at the first beam splitter and travels along a folded path until it is transmitted to the

relay by the second beam splitter. The test surface is sharply imaged onto the camera, and there is also a plane conjugate to the camera in the reference beam. In a Michelson interferometer this conjugate surface in the reference arm coincides with the reference surface, which is imaged sharply onto the camera as well. Hence the shape of the reference mirror is part of the measured map just as much as the shape of the test surface. In the virtual reference interferometer, there is no physical surface conjugate to the camera in the reference beam path, and we call this conjugate plane the virtual reference surface.

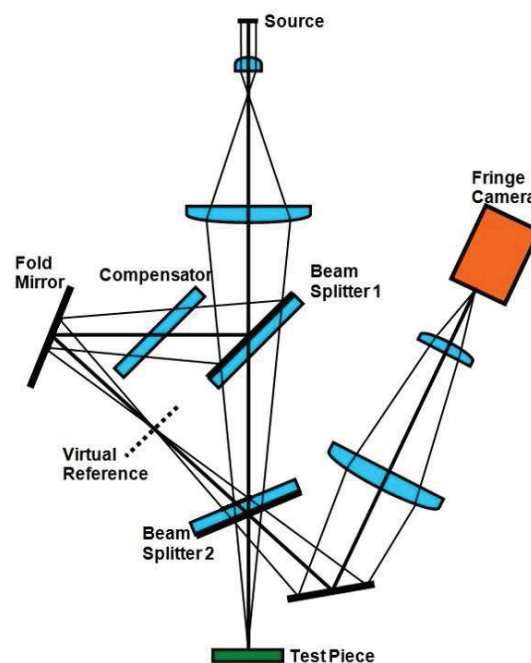


FIGURE 12. Optical schematic of the virtual reference interferometer.

Figure 12 shows the imaging bundle for a given pixel. It has a large foot print on all optical interferometer surfaces. Thus mid and high spatial frequency surface errors of the interferometer are smoothed out, and contribute to the measurement of the test surface only in a very attenuated way. The long-range figure errors of the interferometer optics, however, are not attenuated in the final measurement.

Adding two more plane-parallel compensating windows makes the interferometer fully compensated for a white-light extended source, and we can again operate the interferometer without coherent noise. This was implemented in

the ZeMapper optical profiler in 2011, see figure 13. The virtual reference interferometer has a field of 20x20mm, and is combined with a microscopic profiler channel for smaller fields down to 100x100um. A motorized stage allows for stitching measurements up to 150x100mm in size.



FIGURE 13. ZeMapper with glass disk substrate as test piece.

As implemented, the beam foot print on the optics closest to the virtual reference surface has a diameter of about 3.8mm. From a simple ray-optical smoothing model we obtain the PSD transfer function for the smoothing of the system reference map shown in figure 14.

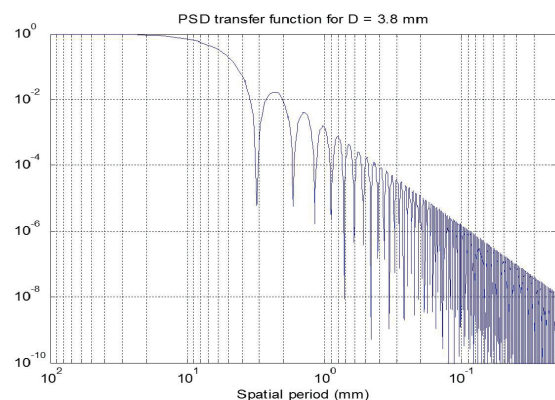


FIGURE 14. PSD smoothing transfer function for the system reference map.

Low spatial frequencies are not attenuated, but for spatial periods of 3mm the PSD attenuation is roughly 20x, and for a 1mm period it is 1000x. Note that since the test piece is in focus, there is no smoothing of its topography in the measured map.

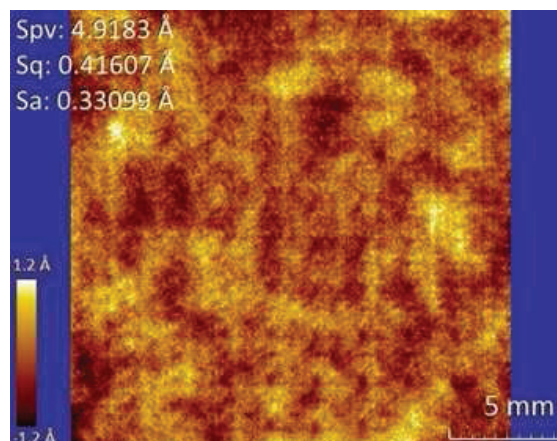


FIGURE 15. System reference map after high-pass filtering with 3mm cut-off period.

The attenuation can be seen in the system reference map after high-pass filtering with a 3mm cut-off, see figure 15. This system reference map was generated by averaging over measurements of a super-polished flat at 6x6 locations. Some periodic artifacts of this calibration process are visible on the maps. Correcting for these artifacts yields an RMS of the system reference map 0.036nm for the 3mm cut-off period, and of 0.015nm for a 1mm cut-off period. The full system reference map without any high-pass filtering has an RMS of 3.7nm. The random RMS waviness noise was measured as 0.02nm (averages of 10) after high-pass filtering with 5 mm cut-off period.

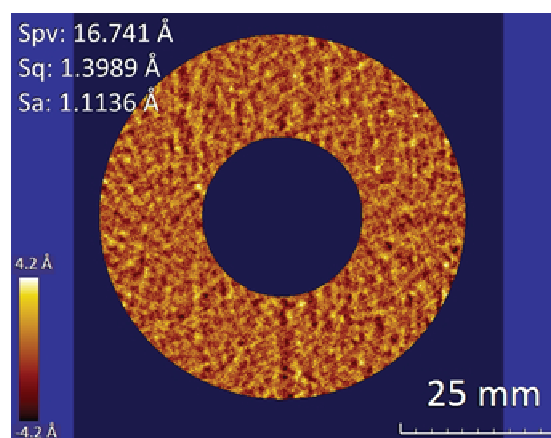


FIGURE 16. Disk waviness map after high-pass filtering with 3mm cut-off. RMS = 0.14nm.

Finally, as application example the measurement result of a 65mm hard disk substrate is shown in figure 16. The map was

acquired with the virtual reference interferometer at 4x4 stitching positions without subtracting a system reference map. After high-pass filtering with 3mm cut-off, the surface RMS is 0.14nm. From equation 3 we see that the optical system reference error of 0.036nm RMS contributes less than 5% to this surface RMS value. Note that faint processing marks are visible on the disk surface with a display scale of +/- 0.42nm.

CONCLUSION

Precision interferometry entails controlling error sources to a high degree. For the three systems presented above the main concern was coherent noise, and in one case also the system reference error. They all use low coherence sources with extended spectral bandwidth and spatial size, such as spectrally filtered halogen lamps or LEDs. With modest map averaging to suppress random noise it is possible to measure the waviness of flats and spheres or aspheres in the Angstrom range.

The availability of coherent laser sources for optical metrology in the 60s revolutionized interferometric testing. The usage of these highly spatially and temporally coherent sources led to simpler instruments for flat and sphere testing where the fringes were no longer localized, thus allowing for great flexibility in the test setup and ease of use. Later the use of computers and phase shifting algorithms provided the ability to routinely obtain quantitative surface maps with sub-nm height resolution.

However, coherent noise represents a fundamental limitation with coherent sources. This was recognized early on, and means to reduce the spatial coherence while maintaining the temporal coherence of the laser source were implemented in commercial interferometers.

The usage of sources with low temporal as well as spatial coherence may appear like a step back, but it still provides the lowest coherent noise in the measurements, especially in the waviness frequency band. Also, suppressing the back-side interference is important in many cases. If the instrument provides good alignment aids, finding the fringes is fast and straightforward. The design of these instruments needs to employ old techniques of coherence analysis and control developed before the advent of the laser, but also incorporates new

developments such as in algorithms, electronics, and light sources.

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Figure measurement of high-aspect-ratio optics by stitching

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INTRODUCTION

We had a goal of designing an interferometer and writing stitching software that could be mounted on a gantry system and used to measure the figure of long, narrow, cylindrical mirrors with very long radius of curvature (ROC) such as those used as synchrotron focusing mirrors. The specifications were to measure a swath about 6 mm wide in 6 mm patches over the length of a mirror whose slope varied from one end to the other by ± 3 mradians without changing the orientation of the interferometer to stay normal to the surface. We describe the design of the interferometer, the approach to stitching and some initial results.

INTERFEROMETER

The interferometer itself was a Nikon 2.5x TI objective and MIPOS 100 PZT phase shifter coupled to a W101 illumination and camera unit made by Opt-E that supports illuminating a 6 mm square field of view and a 2/3", 4 MP camera. The camera is a Point Grey GS3-U3-50S5M with 3.45 μm pixels so the pixel footprint on the mirror is 2.93 μm square. The Kohler illumination source is a nominally 640 nm LED. For prototype purposes the interferometer unit was mounted on a three axis motorized Newmark gantry and the sample was mounted on one axis with 0.1 μm resolution in the scan direction.

SAMPLE MIRROR

The sample used to test out the system was a nominally 20 m ROC, 100 mm diameter spherical silicon mirror so that it had a slope at the edge of the clear aperture of 2.5 mradians. This meant the sag relative to a plane, parallel to the vertex, was 15 μm or 30 μm optical path difference (OPD). This was our first problem because the LED did not have nearly enough coherence to give good contrast over this OPD even though we could resolve fringes with this slope. There was sufficient coherence to get data over about 1/4 of the width of the patch.

SOFTWARE APPROACH

LabVIEW was used to move the stages, PZT to phase shift, capture the video images of the interference fringes and unwrap the phase data. These raw phase data sets were then transferred to Matlab for the reference subtraction and stitching because Matlab is optimized for this sort of calculation.

CALIBRATION OF THE INTERFEROMETER

To produce an initial calibration (with obvious limitations) of the interferometer for software testing we used a 100 mm diameter, $\lambda/20$, Zerodur flat and made 100 measurements of 3 mm overlapping 6 mm patches in an array 25 x 4 to somewhat simulate the geometry of the final stitching. The gantry was used to move over the flat. These 100 measurements were averaged to give a map (Fig. 1) of the reference surface that was then subtracted from each patch of data from the flat. These data for the flat were stitched to give the result in Fig. 2 after passing the data through a spike filter to remove spikes due to surface blemishes (coating pinholes).

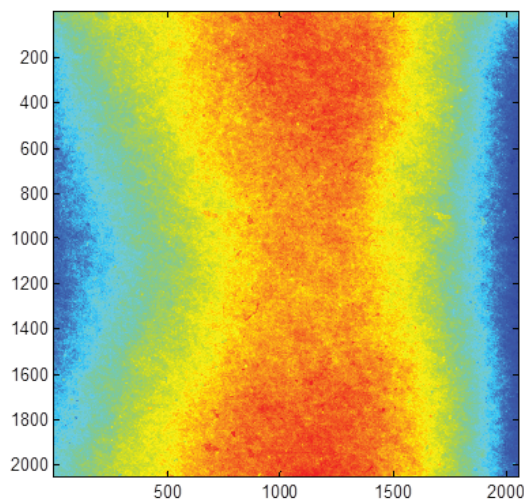


FIGURE 1 Reference surface map, 23 nm p-v, 3.9 nm rms Indices are pixel number

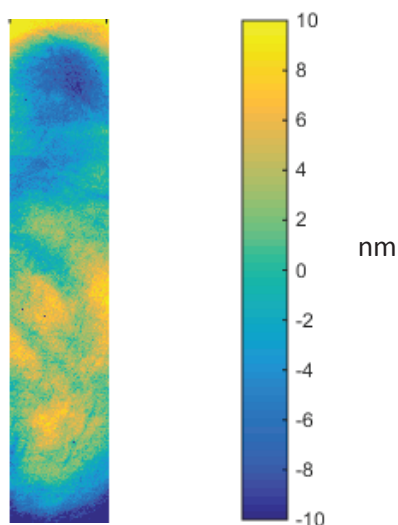


FIGURE 2 *Stitched flat reference surface, 15 x 78 mm area*

The spikes result in missing data. The invalid data was used to create an array of 1's and 0's. A 2D cross-correlation was performed between consecutive measurements to verify stage motion accuracy. We found that the stage met its $\pm 5 \mu\text{m}$ position tolerance, but that led to random position errors of a ± 2 pixels. Matlab was surprisingly, and usefully, fast performing the calculations.

To give a feel for the noise in the stitching operation for the flat data, Fig. 3 shows a single column of data stitched over 78 mm. Since the vertical scale is in nm, the noise on a local level is about 3 nm p-v.

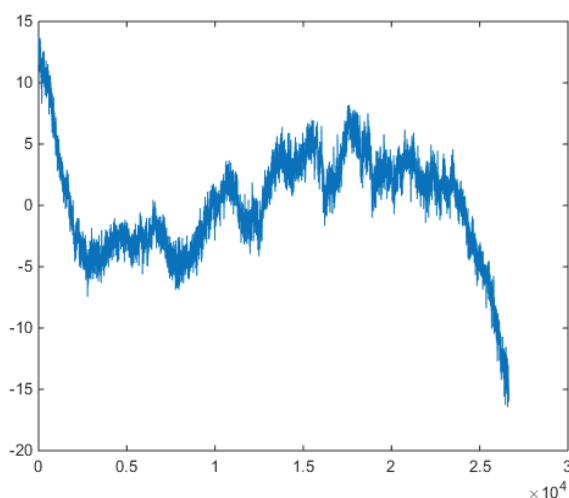


FIGURE 3 *Single column of stitched flat data*

APPROACH TO STITCHING THE SPHERE

Because of the limited coherence length the first stitching was vertical. The number of data sets necessary was determined dynamically by the LabVIEW program. A simple approach of moving vertically, measuring again, until insufficient data was obtained was used. The system moved first one direction, and then back to the starting location and moved the other direction. A running average of valid data at each location was implemented so as to fix the data storage requirements. No reference surface subtraction was performed in this process.

The LabVIEW (32-bit) based program saves the data in a file, and Matlab (64-bit) routines read the data and perform the global stitching operations. This is done because the global fit requires more data space than fits in a 32-bit program. Even so, careful implementation of the Matlab code is necessary to fit most problems in 16 Gb of RAM. Fig. 4 shows the first and last 6 mm patch on the silicon mirror. The Matlab routines also subtract the reference surface error from each map.

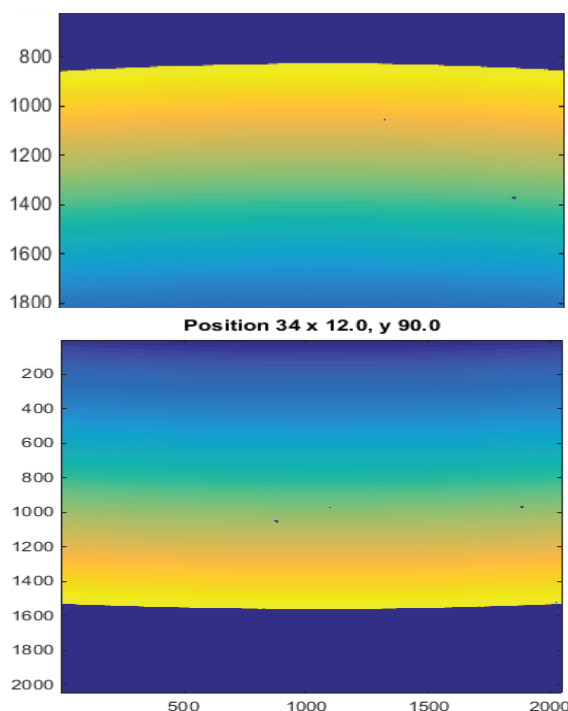


FIGURE 4 *First and last patches in scan of the 100 mm diameter sphere*

After stitching the 34 patches together the overall 100 x 6 mm contour looked like Fig. 5, and shows the overall sag of the sphere of 62.5 μm .

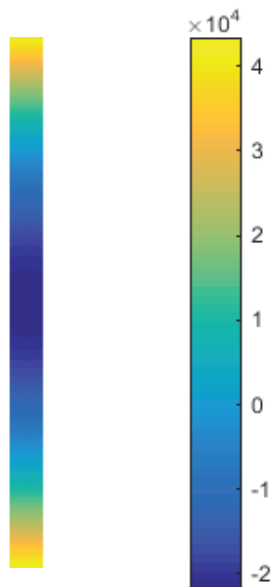


FIGURE 5 The stitched contour with sag of 62.5 μm

We then removed power to see the difference between the stitched surface and the theoretically perfect surface as shown in Fig. 6.

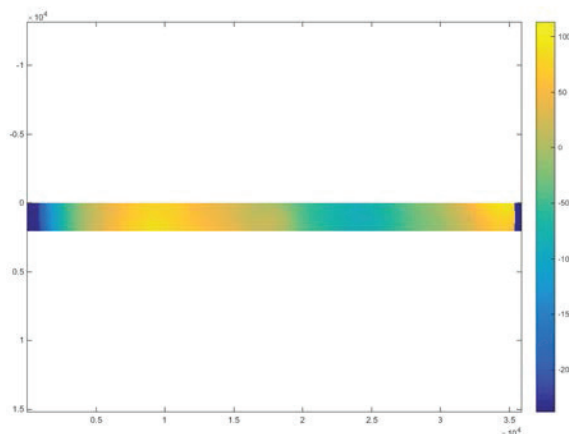


FIGURE 6 Stitched contour with power removed. Left side corresponds to top of Fig. 5. Vertical scale is nm.

A new, nominally identical data set was taken 6 hours later and stitched. The difference between the two sets of stitched data is the red curve in Fig. 7. The local noise was about the same magnitude as that in Fig. 3, but the overall systematic difference was about 50 nm p-v. At

least some of this difference could probably be accounted for by the relatively poor thermal environment in the lab (an office with windows).

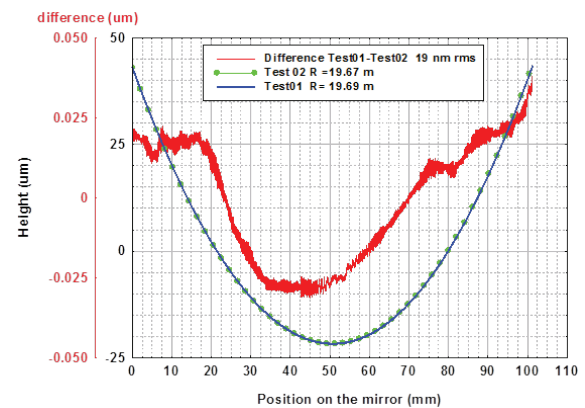


FIGURE 7 Two repeated stitched measurements and their difference (red)

RESULTS AND CONCLUSIONS

From the data in Fig. 5 the sag was 62.50 μm using a wavelength of 635 nm for the LED. When the sphere was measured with a defocused collimator we got a number of 19.834 ± 0.05 m. Using this radius indicates the LED wavelength was about 639.5 nm. Other methods of measuring the sphere were less precise than the defocused collimator measurement. We were encouraged that the results of the stitching matched the radius of the sphere so well. A

The data in Fig. 6 with the power removed shows a residual error of about 350 nm p-v in a shape associated with coma. This error is large enough with respect to the kind of error one would expect to see in a well-polished sphere that it hides the high spatial frequency error one might expect to see such as seen in the reference surface, Fig. 2.

It is also curious that the overall nature of this difference from a perfect sphere and the contour of the stitched flat data are of the same character, roughly comatic in shape. While that error is much less for the stitched flat data than for the sphere, and opposite sign in direction of stitching it gives one concern there may be a systematic error in the way the stitching was done. One expected contributor is distortion in the imaging system. We intend to investigate this subject more thoroughly.

Finally the data in Fig. 7 is encouraging in the sense that the data are only slightly noisier on a

local level than the stitched data from the flat mirror used as a reference. In addition, the repeatability as shown in the overall red curve is far less than the possible systematic error in stitching. If there does turn out to be a systematic algorithmic error, it is easily corrected and the final results will be nearly as good as for the plane surface, something that is easy to measure on a common Fizeau interferometer. On the other hand, to obtain this level of precision on a mirror that is difficult to measure because the curvature is in-between what is easy to measure as a plano and easy to measure with a transmission sphere will be useful to workers in the field.

INTERFEROMETRIC MEASUREMENT OF LARGE SPHERE RADII USING HOLOGRAMS

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INTRODUCTION

Glass lenses and mirrors with spherical surfaces are widely used in optical imaging systems because spherical surfaces can be fabricated efficiently in large numbers with conventional grinding and polishing methods. Spherical surfaces are characterized by their sphericity, the deviation of the spherical surface from a best-fit perfect sphere, and the radius of the best-fit sphere [1]. The radius of a spherical surface can be measured with the well-known radius bench method. In this measurement procedure, the spherical surface is moved in a converging test beam from a position at which the test beam focus is at the spherical surface ("cat's-eye" position) to a position at which the test beam focus is at the center of curvature of the test surface ("confocal" position). The displacement between the two positions is the sphere radius. Low measurement uncertainties can be achieved when laser interferometry is used to measure the displacement and when interferometric wavefront measurements are used to precisely locate the beginning and the end of the displacement [2]. The need to move the spherical test surface by a distance equal to its radius immediately suggests that the method is limited to radii of no more than a few meters. For larger radii, the necessary displacement typically cannot be realized, and uncertainties introduced by air turbulence in the test cavity, and those due to the displacement metrology, increase.

When the radius of a test part is very large it can be estimated by measuring the test part against a reference flat. For intermediate radii of several 10's of meters this is not possible with most interferometers because the resulting spatial fringe frequencies are not transmitted by the imaging optics and they cannot be sampled by the image detector without aliasing. An example are the fringes, shown in Fig. 1, created by a convex radius test plate with nominal radius of 19.13m against a reference flat.

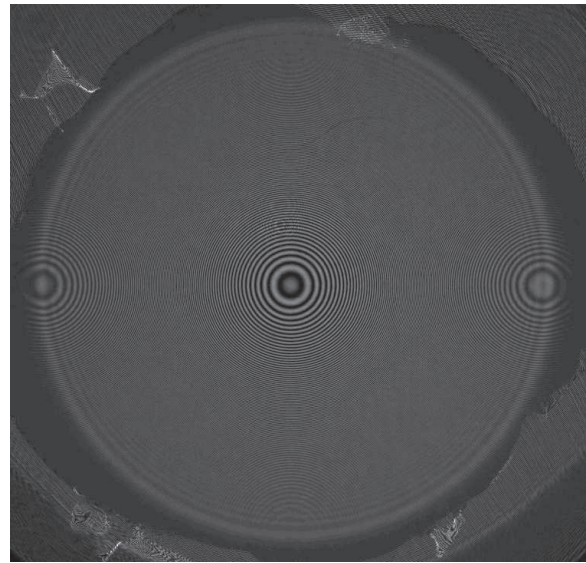


FIGURE 1. Vignetted interferogram of a convex radius test plate with a nominal radius of 19.13m and a reference flat at a wavelength of 632.816 nm.

The radius bench method can be modified for the measurement of large sphere radii by using a pair of spherical test wavefronts instead of a single spherical test wavefront. The test beams are created by a pair of nested zone lenses that are fabricated on a common substrate. One of the test beams is used to create a focus on the test part surface ("cats-eye" position). The second test beam has its focus at the test part center of curvature ("confocal" position). The distance between the two test positions can now be chosen freely during the design of the nested zone lenses. This makes it possible to measure radii of spherical surfaces that are much larger than the length of a typical radius bench with relatively small displacements and short test cavities. In the simplest case the displacement can be chosen to be zero as illustrated in Fig. 2. The resulting nested zone lens then becomes a "test plate", which can be used without a displacement measurement to measure the radius deviation of the

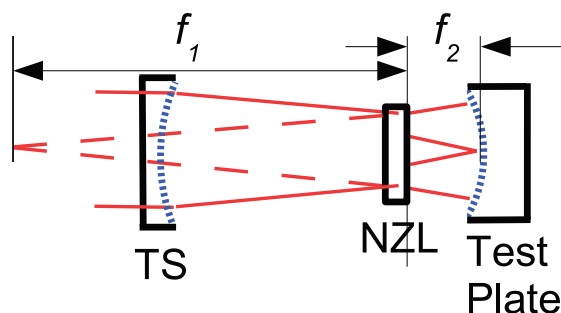


FIGURE 2. Dual-focus nested zone lens (NZL) that is used in conjunction with a transmission sphere (TS) to measure the radius deviation of a spherical surface (test plate).

test part radius from its nominal radius. Uncertainties due to parasitic diffraction orders of the holograms can be reduced or eliminated when one or more of the holograms are implemented as photon sieves [4, 5].

In this article we illustrate the method with interferometric radius measurements of a matched pair of convex and concave radius test plates with nominal radii of 19.13 m. The radius difference of the matched pair of test plates can be evaluated directly with interferometry. Using a matched set of test plates enables a more stringent test of the new radius measurement method than the radius measurement for a single spherical surface [3, 4].

MEASUREMENTS

Fig. 2 and Fig 3 illustrate how the measurements were made. A nested zone lens was set up in the test beam of an $f/7$ transmission sphere. An annular alignment zone mirror (clearly visible in Fig. 3) was used to position the nested zone lens at the correct distance from the reference surface of the transmission sphere and in the correct orientation. The nested zone lenses were designed for the zone lens substrate surface that is facing the radius test plate in the measurement setup (Fig. 3). The central zone lens, which defines the “cat’s-eye” position of the radius measurement, has a diameter of 15 mm and a primary focus at 100 mm distance from the zone lens. It was designed and fabricated as a binary phase Fresnel zone lens. A detail of the zone lens at its center is shown in Fig. 4. The outer zone lens, which defines the “confocal” position, was designed as a binary phase photon sieve [4, 5] to attenuate diffraction orders that contribute to coherent noise. A detail of the photon sieve is

shown in Fig. 5. The measurement setup shown in Fig. 3 shows the convex partner of a matched pair of radius test plates with a nominal radius of 19130.375 mm. The nested zone lenses were designed such that when the focus of the inner zone lens is on the test plate surface, the radius of curvature of the wavefront created by the outer zone lens at a distance of 100 mm from the zone lens is equal to the nominal radius of the test part surface. The radius deviation of the test part can be calculated from the defocus term of a phase measurement in the outer zone of the nested zone lens. Both nested zone lenses were fabricated with a lithography system [6] at the Nano-Structured Optics Laboratory of the National Institute of Standards and Technology (NIST) and the NanoFab of the NIST Center of Nanoscale Science and Technology.

For the radius measurements, the radius test plates were positioned near the focus of inner zone lens so that between 5 and 9 defocus fringes were visible across the diameter of the inner zone lens. The defocus Zernike terms a_2^0 in the inner and outer zone lens areas were then calculated from phase-shifting measurements that were made at a set of part positions along the optical axis (z) through the focus. At each of these positions the phase measurement was repeated five times. In addition to the displacement measurement provided by the defocus of the inner zone lens, the displacement of the test part was also tracked with three displacement measuring laser interferometers that were arranged around the optical axis. Fig. 6 shows that the coefficient of the Zernike defocus term in the inner zone lenses can be approximated as a linear function of the test part displacement with a very small quadratic component that cannot be seen in Fig. 6.

RESULTS AND DISCUSSION

The measured Zernike defocus term a_2^0 for the outer photon sieve as function of the Zernike defocus term for the inner zone lens are shown in Fig. 7 for the concave radius test plate and in Fig. 8 for the convex radius test plate. The defocus term for the outer zone lens at zero defocus in the inner zone lens area are calculated from a linear fit to the data shown in Figs. 7 and 8. In the data for the concave radius test plate in Fig. 7 the expected change in the defocus of the outer zone lens area as a function of displacement is clearly visible. The data for the

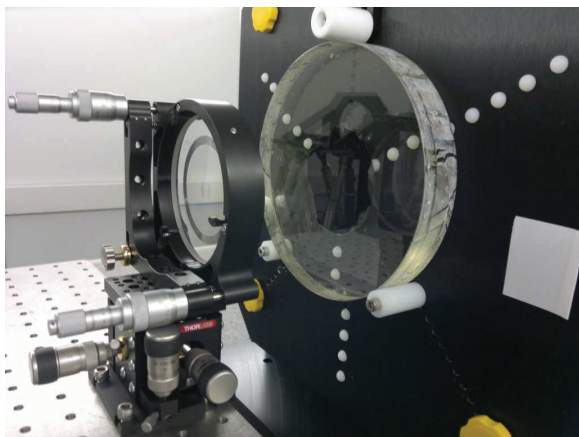


FIGURE 3. Nested zone lens in the beam of an $f/7$ transmission sphere together with a convex radius test plate with 19.13 m nominal radius. The concave and convex radius test plates both have a diameter of about 200 mm.

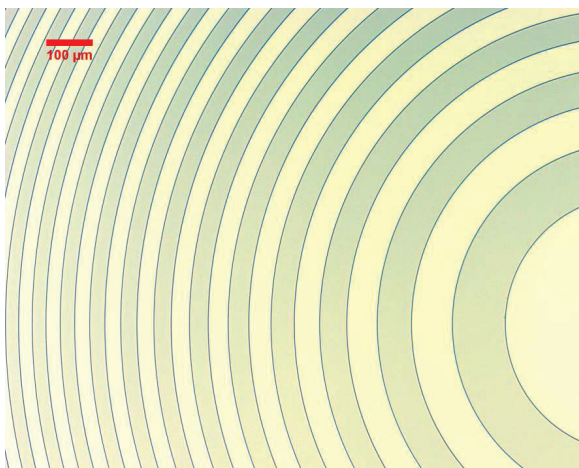


FIGURE 4. Detail of the inner zone lens that provides a focus on the test part surface.

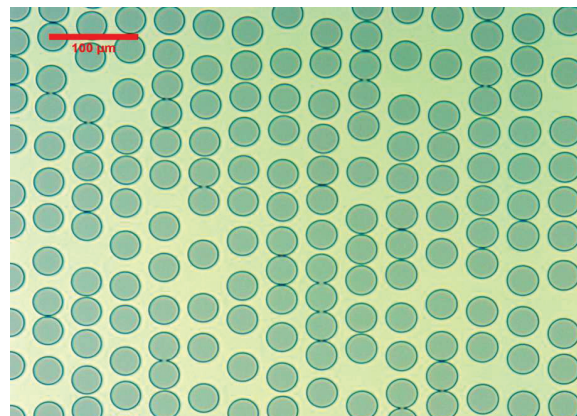


FIGURE 5. Detail of the outer zone lens (photon sieve) that generates a confocal position for the spherical test surface.

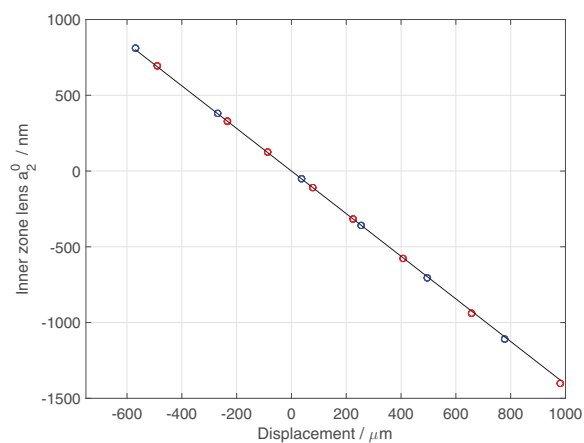


FIGURE 6. Coefficients of Zernike defocus term a_2^0 for the inner zone lens as function of displacement. The blue symbols are the results for the convex radius test plate, the red symbols for the concave test plate. A positive displacement moves the test part towards the zone lens. The solid line represents the theoretical linear approximation based on the zone lens design parameters. Error bars are too small to be visible.

convex radius test plate in Fig. 8 are more noisy. This was caused by a larger amount of stray light from the nested zone lens for the convex test plate due to a mishap during the fabrication of the zone lens. A developer tool failure resulted in over-developing of the exposed photo-resist which caused a line width error, which, in turn, caused a significant amount of light in the 0th diffraction order. The normalization radius in pixels needed for the calculation of the Zernike coefficients was determined from interferograms that were obtained during the phase shifting measurements. The resulting defocus Zernike term for the concave test plate at the focus of the inner zone lens $a_2^0 = -29.63 \text{ nm} \pm 0.05 \text{ nm}$ (for a coverage factor $k = 1$). For the convex test plate we obtained $a_2^0 = 28.64 \text{ nm} \pm 0.2 \text{ nm}$. The uncertainties in the coefficients are standard deviations resulting from the measurement repeatability.

The radius deviation ΔR is calculated from the defocus terms using the following equation:

$$\Delta R = \pm 4a_2^0 \left(\frac{R \pm f_2}{r_z} \right)^2 \pm \Delta z, \quad (1)$$

where R is the nominal (design) radius of the test plate, r_z the normalization radius for the Zernike polynomials, f_2 the focal length of the inner zone lens, and the displacement $\Delta z = 0$ when the test part is positioned at the focus of the inner zone lens [4]. The positive signs in Eq. 1 apply to the convex test surface, the negative signs to the concave test surface. The radius deviation, calculated with Eq. 1, for the concave radius test plate is $47.68 \text{ mm} \pm 0.8 \text{ mm}$ and $47.06 \text{ mm} \pm 0.8 \text{ mm}$ for the convex test plate (coverage factor $k = 2$).

A number of contributors to the uncertainty in the radius deviation were identified:

Zernike defocus term: The error bars shown in Figs. 7 and 8 indicate the standard deviations of the Zernike defocus coefficients for five measurements that were made at each position through the focus of the inner zone lens. From those uncertainties the standard uncertainty for the defocus coefficient at $\Delta z = 0$ was calculated [4]. The uncertainties for the defocus coefficient result in a standard uncertainty for the radius deviation of 0.3 mm

Zernike normalization radius: In our measurements the radius of the unit circle of the Zernike polynomials was determined from

the interferograms that were obtained during the phase measurements through the focus of the inner zone lens. Stray light and diffraction effects increase the uncertainty that can be achieved for the normalization radius to about 1 pixel, which corresponds to $82 \mu\text{m}$. The resulting standard uncertainty in the radius deviation ΔR is 0.26 mm .

Wavelength error: The nested zone lenses were designed for a laser wavelength of 632.816 nm in air. The laser wavelength of the interferometer was measured with a wavemeter and the wavelength could be maintained during the measurements to 5 ppm . The resulting uncertainty in the radius deviation is negligible.

Substrate thickness error: The substrates for the nested zone lenses have a thickness of 12.647 mm . The substrate thickness was measured using a micrometer. The estimated measurement uncertainty of the thickness is $2 \mu\text{m}$. The measured thickness was incorporated into the design of the nested zone lenses. The uncertainty in the radius deviation due to the thickness uncertainty is negligible.

Substrate index of refraction: The substrates were made from a borosilicate float glass. The refractive index is known with a standard uncertainty of about 100 ppm . We measured the refractive index homogeneity and found that it is about 2 ppm . The refractive index uncertainty is not a significant contributor to the uncertainty of the radius deviation.

Scale error of the holograms: Scale errors can be introduced either during the fabrication of the holograms or by fabricating them at one temperature and then using them at another. The scale errors of our lithography system are currently unknown. Significant scale errors due to temperature effects can be ruled out because the nested zone lenses were fabricated and used in environments with a temperature of 20°C that is very tightly controlled.

Diffraction effects: The nested zone lens is located 100 mm outside the focus of the imaging system of the phase shifting interferometer, which results in diffraction ripple at the apertures of the zone lenses. This has a small effect on the coefficient a_2^0 of the

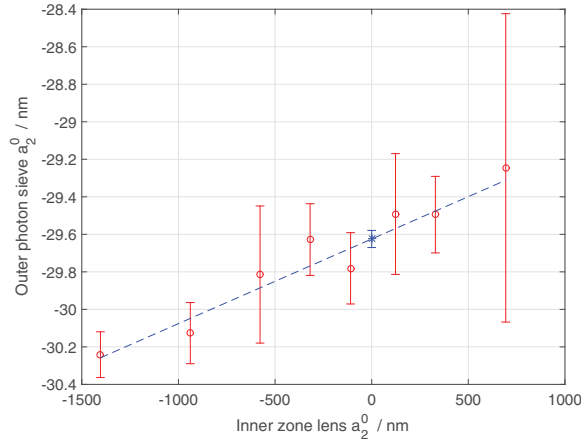


FIGURE 7. Defocus Zernike terms for the concave radius test plate. $a_2^0 = -29.63 \text{ nm} \pm 0.05 \text{ nm}$.

Zernike defocus terms, primarily because it is more difficult to determine the normalization radius for the Zernike polynomials in the measured images.

Limited measurement area: In our measurement the radius test plates have a much larger diameter (200 mm) than the nested zone lens (60 mm). It is important to note that we have only measured a local radius of the test plate near its center, not the best fit radius of the whole surface. Sphericity errors can result in significant variations of the local radius [1].

Hologram alignment: Errors in the position of the hologram along the optical axis result in a test wavefront from the transmission sphere with an incorrect curvature at the hologram. Such position errors occur because the defocus error in the alignment zone mirror is difficult to “null”, because the hologram substrate may be curved, and because the wavefront from the transmission sphere can have sphericity errors. The resulting uncertainty in the measured radius is not yet included in our analysis.

CONCLUSIONS

We measured the radii of a matched pair of radius test plates with nominal radius of 19.13 m using nested zone lenses. Our initial results show that the radius deviations of the matched concave and convex radius test plates agree closely. This suggests that nested zone lenses are a promising interferometric method for the radius measurement of precision surfaces with large radii of curvature.

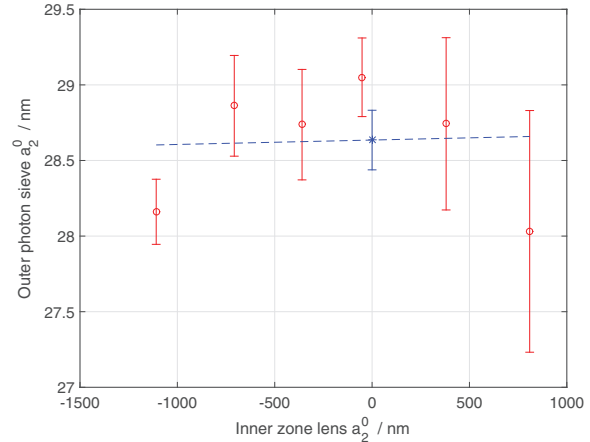


FIGURE 8. Defocus Zernike terms for the convex radius test plate. $a_2^0 = 28.64 \text{ nm} \pm 0.2 \text{ nm}$.

ACKNOWLEDGMENT

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ASPHERE MEASUREMENTS BY CGH – A PARTIAL UNCERTAINTY ANALYSIS

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INTRODUCTION

The two major methods for precision interferometric measurements of aspheres are using stitching interferometry and null optics to transfer the wavefront from an interferometer into an aspheric wavefront. Stitching interferometers are typically limited in their permissible range of aspheric departures and, more importantly (at times) form factor, specifically in part diameter and asphere vertex radius. Commercial stitching interferometers have a maximum cavity length that limits the concave radii that can be measured.

Null optics are typically divided into two categories, those made from a set of spherical optics and the so-called computer generated holograms (CGH). For measurements of macro-optics (i.e. not large telescope mirrors), the null optic(s) is typically placed after the interferometer's transmission sphere (TS) (or flat) and before the test optic as shown in Figure 1. The designed null optic will transform the spherical wavefront from the TS into an aspheric wavefront that theoretically matches the aspheric curvature of test optic. The distances in this setup are specific and errors can result if not correct. Each aspheric null optic setup is designed for one and only one asphere design.

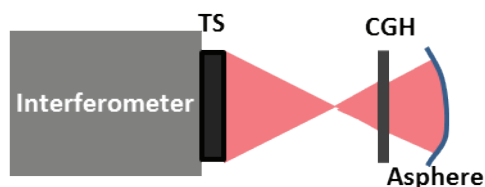


Figure 1: Measuring an asphere with a CGH with a commercial interferometer.

CGHs are a diffractive optical element, typically printed chrome on a fused silica photomask blank. CGHs can have increased flexibility over spherical null optics and have an increased design range.

Using CGHs are never as easy as described in the literature. The initial alignment is a struggle in the dark searching for return spots. With time, though, alignment becomes easier and putting the CGH in becomes almost as easy as switching a transmission sphere.

CGH SYSTEM ERRORS

Next, the system errors must be considered. Methods for measuring the interferometer system errors are well established such as the three-flat test, random ball test, n-position test, and the shift-rotation test. We have not seen these calibration methods applied to CGH use. Typically errors due to the CGH blank are removed post measurement, in software using the following procedure.

First, the transmitted wavefront error of the CGH is measured in a plano/plano configuration (or this information is supplied by the CGH manufacturer). The TWE error of the CGH (which of course, must be corrected for any errors in the plano/plano cavity) is then typically represented by a 36 or so Zernike term representation¹. This TWE is then converted in software (optical modeling or mathematically) to the at-use configuration, typically in a converging beam. The error in this conversion will be increased in systems with fast transmission spheres and with the thickness of the CGH.

Separately, the error in the transmission sphere must be measured using any of the standard calibration methods (should be done at the approximate same focus as for the measurement). The TS error must then be masked to the 'use area', which is an interesting problem, especially for an off-axis design CGH. This TS error is also typically represented with the 36 (37) term Zernike polynomial fit.

¹ There are many representations of the Zernike polynomials. For simplification purposes we use the non-normalized 'Fringe' Zernikes.

The masked TS and CGH transmitted wavefront Zernike polynomials errors are then summed and the final system error is determined. This system error can then be subtracted from each subsequent measurement.

In practical use, this method produces unacceptable high levels of uncertainty, practically for tightly specified aspheres $<\lambda/20$. The major reasons that this method is unacceptable are:

- that a 36 term Zernike does not well represent the errors in CGH transmitted wavefront errors;
- the conversion from the CGH TWE plano/plano test to the at-use configuration; and
- the masking of the TS error is questionable, especially for off-axis designed CGHs.

For these reason, we moved to using an n-position test to the CGH asphere measurements. The n-position test, while time consuming, is an in-situ measurement of the system errors (expecting rotationally symmetric errors). This removes much of the uncertainty in applying ex-situ measurements of the system error. The theory behind the n-position position appears to hold for the more complicated CGH system, but alignment errors with the axis of rotation become much more problematic as described here.

In addition, applying the n-position test appears to allow us to measure (to some level) the coma in the asphere due to wedge and decenter in the part. With this measurement, we can correct the surface to correct errors in the wedge and decenter.

CGH N-POSITION TEST

Evans and Kestner showed that the n-position [1] test can measure the system error of an interferometric measurement. The n-position test measures the “nonrotationally symmetric terms of all angular orders except CN_0 ” where C is a positive integer. In practical terms, the n-position test cannot measure the rotationally symmetric errors (power, spherical, etc.). And, to measure higher order errors, more positions must be taken.

An n-position test is accomplished by rotating the part around the optical axis n times and averaging the resultant measurements. After

removing the erroneous rotationally symmetric terms and CN_0 terms, the system error is calculated. This system error can be subtracted from any subsequent measurement to determine the true error in the part. The rotationally symmetric terms must be considered separately (an n-position test can be combined with a 3-sphere test to fine the rotationally symmetric terms). With enough positions, the CN_0 terms can typically be neglected.

There are some practical considerations to applying the n-position test to a CGH asphere test. First, as we show below, the alignments of part's axis to the rotation axis and the optical axis are critical. This completely eliminates the ability to use a horizontal interferometer. In addition, because we have found the CGH blanks can have higher-order errors, many rotational measurements will be required (up to every 12 degrees for a 30 position test). This necessitates a vertical interferometer with a positioning spindle. In addition to motor control on the spindle, control on the other positioning axes of the system is advantageous. We have a system for motor control of the x,y of the interferometer, tip/tilt, rotation and z of the part, and x,z of the CGH. The remaining adjustments of the CGH (y and tip/tilt) are accomplished using manual control.

MEASURING COMA

Measuring the coma in an asphere measurement is always difficult, particularly for CGH asphere measurements. Coma will be caused by any tilt or lateral misalignment in the system between the TS and the CGH and between the CGH and the asphere. This begs the question of why not remove the coma in the measurement? This is a short-sighted response. There can be actual coma error in the surface that can be real. In addition, coma can be a sign of large wedge and decenter error between the two surfaces of the optic. Using an n-position test allows us to measure and correct coma due to wedge and decenter (to a certain level). Note that the n-position test (if the part is well aligned to the rotation axis) will calculate CGH coma errors due to misalignment as part of the system error. Thus, misalignment coma errors will be removed from the measurement.

In most cases, when we use an interferometer for measurement, we only consider the surface under test and disregard the back surface and edge of the optic. Wedge between the two

surfaces are typically considered offline with standard indicator measurements on an air-bearing spindle. Separating these two measurements (surface error and wedge) is problematic especially for aspheres where wedge can quickly lead to a large amount of error that shows itself as surface coma error.

With the setup that we have built for the CGH n-position test, we can take the wedge and decenter of the part into account to correct the surface for any wedge and decenter without worry that alignment errors in the setup affect the measurement of the coma due to wedge.

This is accomplished by setting up the part and aligning the spherical back surface and/or part diameter to the rotation axis (which was already aligned to the optical axis). The alignment of the spherical back surface is accomplished by taping the part and measuring the surface runout with an indicator as the part is rotated. If both the edge and back surface must be measured to, the spherical back surface is adjusted using tilt and the edge is moved laterally using taping. The precision of this alignment will be considered.

Once the back surface and edge of the part are aligned to the rotation axis, the full n-position test is done. The system error will show some coma due to the system alignment. But, the coma remaining after subtracting the system error is due to tilt and wedge error of the part. This error may be large in the initial stage of manufacturing and it can be corrected using sub-aperture polishing. We have found success in using this method to correct coma due to wedge and decenter in the part.

Another method to correct this error would be to measure the wedge offline and calculate the theoretical coma required to cause that wedge and polish in the opposite coma. The method shown here appears to be better as it is a direct measurement of the coma.

This method will work to a certain level, but as the coma error becomes smaller, the ability to position the part because a dominant source of error. This is described below.

POSITIONING SIMULATIONS

A major source of uncertainty in the n-position test with CGHs is the ability to position the rotation axis co-incident with the optical axis. The analysis presented here is for the simple case of a lateral offset error of the asphere. This analysis will show that even for a perfect system, asphere alignment can affect the system error significantly. Not considered here, but relevant is the effect of offset (misalignment) when the system error is non-perfect.

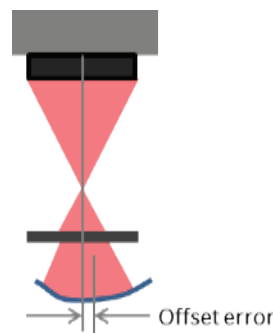


Figure 2: Offset error of the asphere CGH measurement.

An asphere is typically defined with the standard asphere conic equation:

$$z = \frac{cS^2}{1 + \sqrt{1 + (k + 1)cS^2}} + AS^4 + BS^6 + CS^8 + DS^{10} \dots, \quad S = \sqrt{X^2 + Y^2} \quad \text{Eqn. 1}$$

where z is the surface sag, c is the curvature, S is the lateral rotationally symmetric position, k is the conic constant, and the coefficients of the higher order terms are A , B , C , and D . Other asphere definitions are possible.

To determine the effect of a lateral offset of the asphere on the system error calculated using the n-position test, we ran a simulation as described here. First, the 'perfect' wavefront from the asphere is simulated by solving the asphere equation with the parameters of interest. Then, the offset part is solved for using the same equation with an offset value in the x parameter. The measurement is simulated by taking the difference between the perfect wavefront and the shifted wavefront. Note that the analysis presented here was performed on

spheres with infinite radii (plano) to remove the complication of calculating the normal error instead of the sag error. The magnitude of the error should correlate.

The n-position test was simulated by rotating the offset part (about the original central axis) the n times (here, 24, every 15 degrees). A measurement (difference) was taken at every position. The system error is then the average of all the simulated measurements. The following figures show the result of these simulations when the asphere coefficients are considered independently and the offset is varied. In addition, we also show the error when the offset is help constant and the asphere coefficient is varied. Figure 3 through Figure 5 show the errors in the n-position test with a perfect system but a lateral offset of the asphere of 0.001% (1 μm), 0.005% (5 μm), 0.01% (10 μm). The dominate error is Zernikes 6, 13, and 22 (coma² and it's higher orders). As expected, the higher order asphere coefficients cause higher order errors to be relevant, but in general, the higher order aspheric terms are not significant because the magnitude of error is less than < 0.00005 wv (632.8 nm = 1 wv) for a typical part.

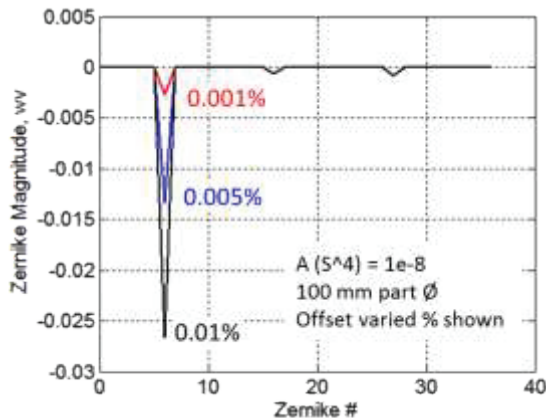


Figure 3: Error in the n-position test result with a perfect system error but lateral offset of the asphere of 0.001% (1 μm), 0.005% (5 μm), 0.01% (10 μm) with only an A term in the asphere definition ($A = 1e-8$).

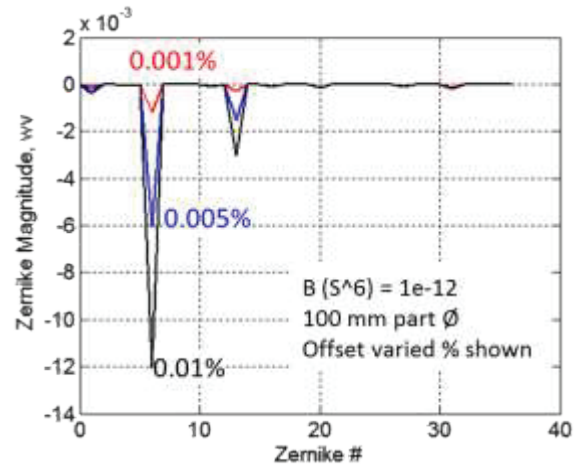


Figure 4: Error in the n-position test result with a perfect system error but lateral offset of the asphere of 0.001% (1 μm), 0.005% (5 μm), 0.01% (10 μm) with only an B term in the asphere definition ($B = 1e-12$).

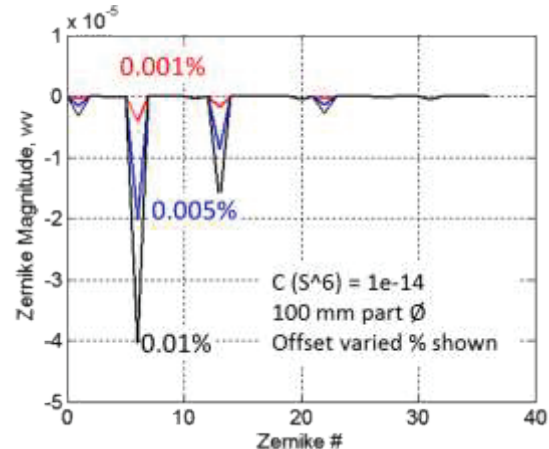


Figure 5: Error in the n-position test result with a perfect system error but lateral offset of the asphere of 0.001% (1 μm), 0.005% (5 μm), 0.01% (10 μm) with only an C term in the asphere definition ($C = 1e-14$).

The more significant error is in the lower order aspheric coefficients where a positioning error of 10 μm (0.01% of a 100 mm diameter part) can lead to 0.025 wv of coma error, which is large in magnitude for many applications. The error will increase with an increase in the aspheric coefficient as shown in Figure 6 where the Zernike coefficient error is approximately linear increasing with the aspheric term.

² Z6 (coma) = $(3\rho^2 - 2)\rho\cos\phi$

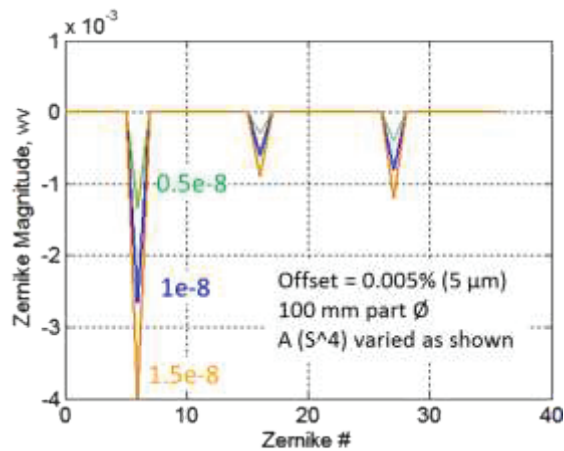


Figure 6: Error in the *n*-position test result with a perfect system error but lateral offset of the asphere of 0.005% (5 μm) with varying the *A* term in the asphere definition.

ANALYSIS

The results shown above seem to show that extremely precise (literally infeasible) positioning of the asphere must be done to use the *n*-position test for a CGH asphere measurement. But, we must take into account that we are permitted to have some wedge and decenter in the asphere. This specification accounts for the ability of the customer to adjust the optic in their system.

The better response is that coma (and its higher orders) should be measured and corrected for with the *n*-position test down to a certain level. The level should be determined from the wedge specification and from the simulations with the asphere definition. For example, with a specific asphere and a wedge specification of 2 μm , we can show that 2 μm of decenter would result in 0.02 wv of coma error using simulations of the CGH measurement. With this, we know that we should not try to correct less than 0.02 wv of coma error. Knowing that we can disregard coma less than 0.02 wv, we can determine the positioning accuracy needed. For example, for the asphere in Figure 3 we would need better than 10 μm of positioning accuracy, but 5 μm would be acceptable, which is achievable with an indicator.

More to Consider

The analysis presented here is limited in its scope. Much more would have to be considered for a full uncertainty analysis, partially in how

specific system errors are affected by offsets of the asphere.

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Comb Calibrated FMCW LADAR for Ranging and Surface Mapping

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INTRODUCTION

Laser detection and ranging (LADAR), since its inception in the 1960's, has become a tool of choice when it comes to surface profiling and mapping in general. Compared to direct energy or incoherent LADAR, which directly measures time-of-flight of a laser pulse off of a target with a photodiode, coherent LADAR, including frequency-modulated continuous wave laser (FMCW) LADAR, can provide much higher sensitivity through optical heterodyne detection and higher resolution, precision, and accuracy through high optical bandwidths [1]. FMCW LADARs have achieved range resolutions well below 1 mm and range accuracy well below 1 μm [1–5]. Furthermore, the heterodyne nature of FMCW LADAR allows for very sensitive detection of weak signals scattered from rough surfaces at a distance. This characteristics make FMCW LADAR a competitive surface profiling and imaging system.

Here we present a FMCW system with a MEMS based frequency-swept laser, with center wavelength of 1560 nm. The frequency sweep is driven over an optical bandwidth of $B=1$ THz by a 1 kHz quasi-sinusoidal waveform, giving one range measurement every 0.5 ms. To compensate nonlinearities in the laser's frequency sweep, its instantaneous frequency is constantly measured relative to a free-running frequency comb. Together with real-time processing, this approach allows to run the laser at its maximum update rate, which is crucial to obtain 3D surface profiles, and still retain a bandwidth limited range resolution of $\Delta R=c/2B=150$ μm .

SYSTEM DESCRIPTION

An overview of the FMCW LADAR setup used for 3D imaging is seen in FIGURE 1a). The light from a frequency swept laser is split into a measurement and a local reference path. Objects (here a flower) are scanned with a fast steering mirror (FSM). Simultaneously a small portion of the swept laser's output is beat against a mode-locked laser (a free running

optical frequency comb) and detected in phase and quadrature (I/Q) to extract the instantaneous frequency used to calibrate the frequency sweep. FIGURE 1b) shows a block diagram of the data processing, which is done on a field programmable gate array (FPGA).

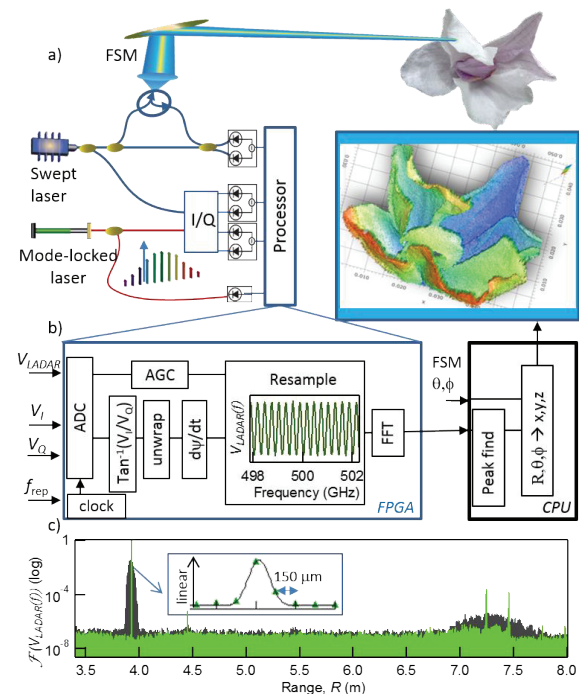


FIGURE 1. a) Overview of setup, as described in the text. Blue/red lines: polarization maintaining fiber. b) Block diagram of processing as carried out on a FPGA. The processing is clocked by the mode-locked laser. AGC: automatic gain control. After extracting the radial range, it is converted (together with the FSM angle readings) to Cartesian coordinates, resulting in a 3D point cloud. c) Linearized 'range spectrum' as discussed in the text. One specific range measurement is extracted as the center of mass of the peak, here just below 4 m. Some spurious reflections are visible just below 7.5 m; they originate among others from the circulator used to couple the light in and out of the free-space section. The black trace shows a distorted range

spectrum obtained assuming the frequency sweep waveform is a pure sine wave. The inset shows the FPGA output, consisting of 8 points across the range peak, with a spacing of $\sim \Delta R = 150 \mu\text{m}$.

Linearization of non-linear frequency sweep

Here we describe the linearization of the frequency sweep in processing following Ref. [5]. The swept laser field is $E(t) = |E|e^{2\pi i\phi(t)}$ and the heterodyne voltage between the local reference and the return signal is then

$$\begin{aligned} V_{\text{LADAR}}(t) &\propto \cos[\phi(t) - \phi(t - \Delta t_{\text{obj}})] \\ &\approx \cos\left[\Delta t_{\text{obj}} \frac{d\phi(t - \Delta t_{\text{obj}}/2)}{dt}\right] \quad (1) \\ &= \cos\left[2\pi\Delta t_{\text{obj}} f_{\text{laser}}(t - \Delta t_{\text{obj}}/2)\right] \end{aligned}$$

where Δt_{obj} is the time delay to the surface and back (with respect to the local reference branch). The swept laser frequency $f_{\text{laser}}(t) = f_0 + \alpha t$ could be directly substituted in Eq. (1) for a perfectly constant sweep rate α to extract Δt_{obj} and hence the range measurement directly from the measured $V_{\text{LADAR}}(t)$ (typically through a Fourier transform with respect to time of $V_{\text{LADAR}}(t)$).

For our FMCW LADAR, in which the laser is swept at its maximum rate, the frequency sweep is not linear and we directly measure $f_{\text{laser}}(t)$. This measured frequency sweep is then substituted into Eq. (1) and the Fourier transform is taken with respect to $f_{\text{laser}}(t)$ instead of time, which gives the range spectrum

$$F(R) = \int_{-B/2}^{B/2} V_{\text{LADAR}}(f_{\text{laser}}) e^{-j4\pi f_{\text{laser}} c^{-1} R} df_{\text{laser}} \quad (2)$$

where the integral is across the laser's optical sweep bandwidth B and we have rewritten the time delay in terms of the range given as $R = \Delta t_{\text{obj}} \cdot c / 2$, where c is the speed of light in air. Higher-order terms in the Taylor expansion of the phase lead to negligible errors [6], however it is crucial that $f_{\text{laser}}(t)$ is measured simultaneously with $V_{\text{LADAR}}(t)$.

The resulting spectrum $F(R)$, as seen in FIGURE 1c), has a peak with a width of the range resolution

($\Delta R = c / (2B)$) and is centered at the range to the surface (with respect to the local reference delay).

The linearization of the frequency sweep [4,7–9] is hence obtained against the perfectly spaced teeth of the free running frequency comb in the signal processing stage of the range measurement. Recording the instantaneous phase between the swept cw laser and the comb teeth and counting the frequency comb's repetition rate against a rf-standard allows to calibrate the measured range against an rf standard.

PRECISION WHILE RANGING TO A SINGLE STATIC POINT

In a first experiment the FMCW LADAR is set up to range to a single specular target (collimated beam and glass wedge) [4]. FIGURE 2 shows the range measurement, as obtained between 2 glass wedges separated by up to one meter on a rail. To evaluate the range measurement accuracy, the relative distance between the glass wedges is simultaneously measured with a HeNe interferometer. Both measurements agree to within less than 100 nm across the 1 m measurement range. This accuracy is ultimately limited by the knowledge of the air refractive index [10].

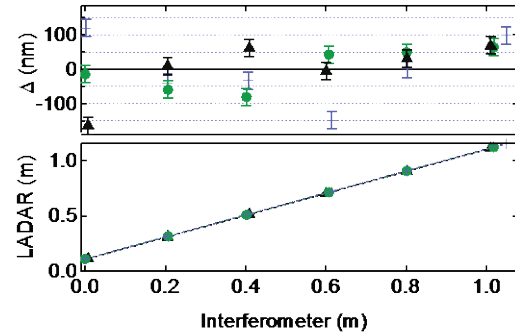


FIGURE 2. Ranging to a specular reflection over 1 m compared to a HeNe interferometer. Top panel: difference between both measurements, showing very good agreement.

FIGURE 3 shows the precision limitation given by the signal-to-noise ratio (SNR). FMCW LADAR has a statistical limit of $\Delta R / (\text{SNR})$. In our system a precision below 10 μm is maintained down to return powers as low as $\sim 75 \text{ fW}$, which corresponds to a signal attenuation of $\sim 110 \text{ dB}$ relative to the launched optical power of 9 mW.

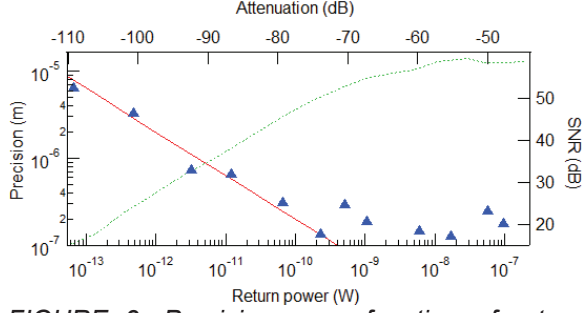


FIGURE 3. Precision as a function of return power to a diffusely scattering target (brushed Al-plate), ranging to a single point. The measurement shown in blue is the average of one up and one down sweep to mitigate Doppler effects. The red line indicates the theoretical precision limit of $\Delta R/\text{SNR}$. Green is the measured SNR of the range spectral beat.

FUNDAMENTAL PRECISION LIMIT FOR LASER RANGING TO A DIFFUSELY SCATTERING TARGET: SPECKLES

While ranging to a surface with roughness σ_z , any coherent LADAR system with wavelength $\lambda < \sigma_z$ is subject to speckles. Speckle is the far-field intensity pattern produced by the mutual interference of a set of random wave fronts [11]. In our system the limiting factor is speckle phase noise [12]. It can lead to large 'range outliers' when measuring range while scanning across an otherwise flat surface, as seen in FIGURE 4b).

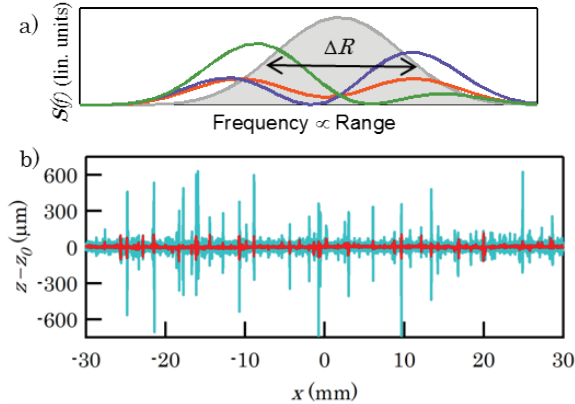


FIGURE 4. a) Measured range spectrum. In the absence of speckle the signal has a width linked to the resolution of the system (grey). Occasionally range spectra exhibit a strong phase-modulation due to destructive interference in the speckle field (colored traces). b) Dependence of range outliers from speckle phase noise on optical bandwidth while scanning across a flat surface in x-direction. Measured range z minus mean range z_0 to a flat metal piece for $\Delta R \sim 150 \mu\text{m}$ [$B = 1 \text{ THz}$] (red)

and $\Delta R \sim 1.5 \text{ mm}$ [$B = 0.1 \text{ THz}$] (blue). The range precision (standard deviation) is $6.5 \mu\text{m}$ ($B = 1 \text{ THz}$). The apparent outliers are a result of strong speckle phase modulation. (Colored traces in (a)).

The range distribution, while mapping a mechanically flat surface with roughness σ_z does not follow a Gaussian distribution as one would expect from the surface roughness, but [12,13]

$$P_T(\delta z) = k \frac{\sigma_z^2}{2(\sigma_z^2 + \delta z^2)^{3/2}} \text{ for } |\delta z| < \Delta R/2 \quad (3)$$

$$P_T(\delta z) = 0 \text{ for } |\delta z| > \Delta R/2$$

where $\delta z = z - z_0$ is the error in the measured range from the true value z_0 , σ_z is the one-standard deviation surface roughness (assumed to be Gaussian), and k is a normalization factor. The truncation is given by the intrinsic resolution of the system. This truncation can also be understood by a maximum possible phase jump of π occurring during a frequency sweep, due to speckle destructive interference (as is the case for two scatterers). The maximum change in phase slope is then $\delta\theta \sim \pm\pi/\tau$, which gives a maximum range error of $\delta z = \pm c/(4B) = \pm \Delta R/2$. This analytical model is fitted to a range measurement distribution in FIGURE 6b).

The speckle-induced precision limit is the standard deviation of the above distribution [12]:

$$\langle \delta z^2 \rangle \approx \sigma_z^2 \{ \ln(\Delta R/\sigma_z) - 1 \} \text{ for } \Delta R/\sigma_z \gg 1 \quad (4)$$

For a $3\text{-}\mu\text{m}$ surface roughness with $B = 1 \text{ THz}$, for instance, $\langle \delta z^2 \rangle \approx 3\sigma_z^2 \approx (5\mu\text{m})^2$, corresponding to an excess variance of $2\sigma_z^2$.

Speckle are hence determining the fundamental system performance for a high-bandwidth, high SNR coherent LADAR. This effect is significantly reduced with higher optical sweep bandwidth, which determines the truncation in the range distribution. Therefore higher optical bandwidth gives better 3D imaging quality, although for very different reasons than expected based on the statistical, SNR-related uncertainty.

OVERALL UNCERTAINTY/ PRECISION

TABLE 1 summarizes and quantifies the error sources of the here presented FMCW LADAR system, including references describing the effects in more detail. The uncertainties are stated at a stand-off of 10 m. Summing the individual uncertainties in quadrature, an overall

uncertainty of $9.5 \mu\text{m}$ is calculated for a 10 m stand-off with a comb-calibrated FMCW LADAR applied to 3D imaging.

TABLE 1: error budget with individual uncertainties σ

$\sigma (\mu\text{m})$	Source of uncertainty
2	Comb offset frequency drift [4]
2	Timing offsets in FPGA processing [5]
3	Signal-to-Noise limitations [5]
4.3	Speckle-induced phase noise [12]
<1	Speckle-induced frequency noise [12]
1	Overall path length drift [5]
7	Spherical to Cartesian calibration [5]
1	Doppler (target motion < $10 \mu\text{m/s}$) [4]
= 9.5	Total

3D IMAGES AT A STAND-OFF

In this section we show 3D point clouds as obtained with our comb-calibrated FMCW LADAR. The fast steering mirror, as seen in FIGURE 1a), scans over the to be mapped object. There is a tradeoff, as described in Ref. [12], between the speed of this lateral scan and additional noise due to conversion from speckle phase noise to frequency noise, which in turn affects the covered surface in a given time. The system acquires one pixel in 0.5 ms, resulting in a 8.3 min acquisition time for a 1 MPixel image.

FIGURE 5 shows the photo of a machined step block in panel a) and the measured 3D point cloud in b). Shown in c) is the average of 30 cross-sections in the x-direction from the FMCW LADAR image (green trace) compared to an average of 12 measurement points taken with a coordinate measuring machine (CMM, blue crosses). Unlike LADAR, the CMM is a contact measurement with a mechanical probe. The inset shows an expanded view of the first 3 steps of the machined block; note that the height of the second step is only $\sim 30 \mu\text{m}$. The difference (error) between the LADAR and the CMM (red crosses) has a standard deviation below $2 \mu\text{m}$.

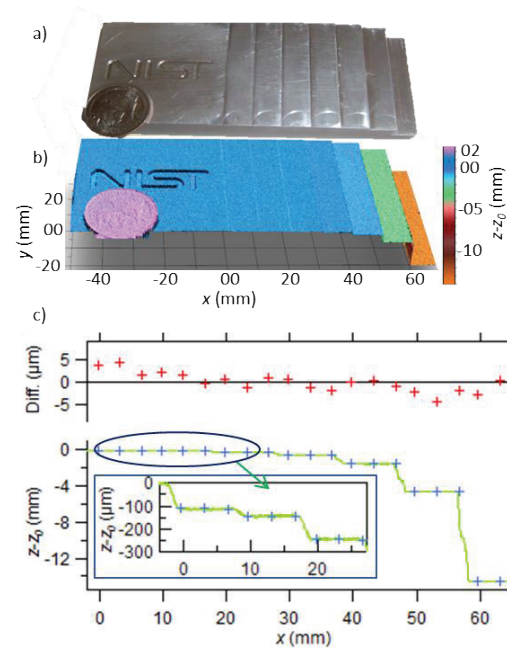


FIGURE 5. a) Photo of machined NIST step block. b) False colored 3D point cloud as acquired at a standoff of $z_0=4.76 \text{ m}$. c) Comparison between a CMM measurement and the FMCW LADAR measurement.

In FIGURE 6a) the 3D point of a flat granite surface is seen. Even though mechanically flat, optically the granite is a porous conglomerate and the LADAR measures additional reflections from underneath the mechanical surface. In b) the measured range distribution is plotted in blue. Black is the analytical model as given by Eq. (3). The left part (upper surface of the plate) of FIGURE 6b) shows good agreement between measurement and model, however additional reflections lead to a double exponential tail to the right, extending up to 1 mm into the granite.

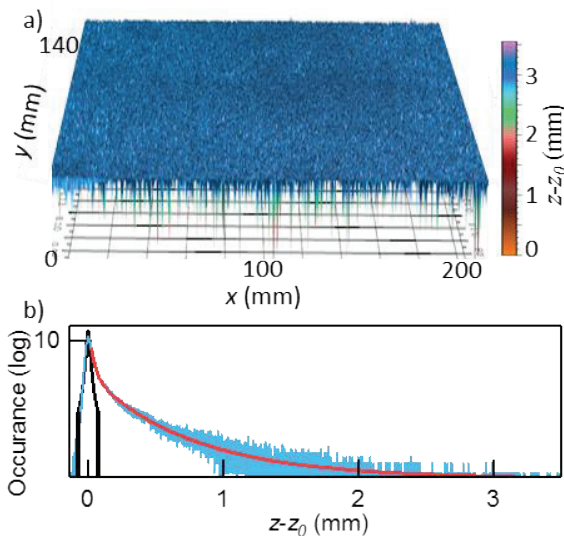


FIGURE 6. a) 3D point cloud of a mechanically flat surface (granite) as measured at a stand-off of $z_0=10$ m, b) Measured range distribution (blue) with a fitted double exponential decay (red), indicating measurements to scatterers underneath the surface. Black: analytical model of the distribution.

POSSIBLE APPLICATION: FORENSICS

In forensic science, plaster casts of impression evidence are still a widely used technique [14–16]. They are labor intensive, difficult to cross-reference, and can destroy the evidence especially on soft ground.

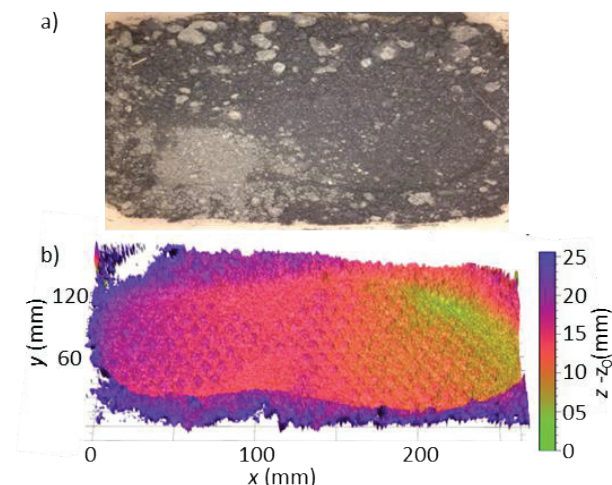


FIGURE 7. a) Photograph of a footprint in dirt, acquired at close range. b) Calibrated 3D surface image of the same footprint in dirt acquired at $z_0 = 10.568$ m stand-off. This image contains 1.12 Mpixels and reveals more detail than the photo in a).

FIGURE 7 and FIGURE 8 show 3D surface profiles taken with the comb-calibrated FMCW LADAR at 10.5 meter stand-off distance. FIGURE 7, panel a) shows a photograph of a shoe impression in dirt, and panel b) shows the corresponding false colored 3D point cloud, revealing significantly more detail about the impression and the structure of the dirt compared to a). FIGURE 8b) shows the tread of the actual shoe measured at the same 10.5 m stand-off distance. The aluminum tabletop and the shoe sole rubber are captured in a single measurement without the need to adjust any optical power levels, demonstrating the wide dynamic range in return intensity of FMCW LADAR. The shoe tread shows individual wear marks from a bicycle pedal. Such marks are important to capture since they can link a specific shoe to a crime scene.

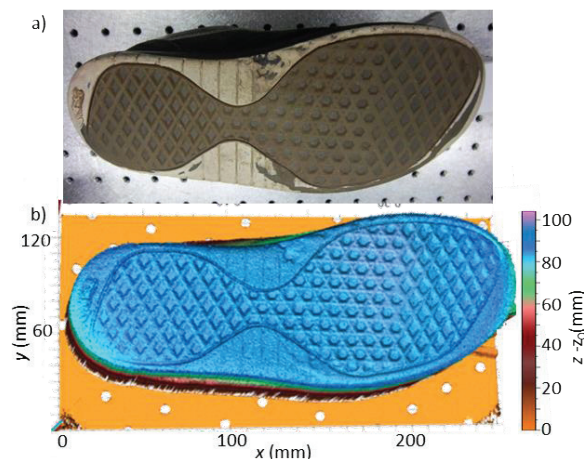


FIGURE 8. a) Photograph of the tread of the shoe used FIGURE 7. b) Calibrated 3D image of the shoe tread revealing the crests and valleys of the rubber sole, along with the worn mark of a bike pedal, acquired at $z_0 = 10.587$ m stand-off.

CONCLUSION

We developed a comb-calibrated FMCW LADAR that can be applied to 3D mapping of diffusely scattering surfaces at over 10 meters of stand-off. There is no need for repeated calibrations, since the swept laser frequency is continuously monitored with a free-running comb. The accuracy can then be tracked to a rf standard via a count of the frequency comb's repetition rate, and the overall achieved uncertainty is at the ppm level. The linearization of the nonlinear frequency sweep and the range processing is carried out in real time in an FPGA. The system offers a high sensitivity and offers a wide dynamic range, achieving the same precision across 70 dB variation in return power. We provided some example calibrated

3D images that reveal small details captured in ambient lighting at 10 meter-scale stand-offs. This system is potentially useful for various applications, such as precision machining, precision assembly of components or forensics.

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Super-localized swept wavelength interferometric ranging using frequency estimation methods

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INTRODUCTION

Coherent swept wavelength interferometry (SWI) is a powerful measurement tool, unique in its ability to simultaneously resolve distances between multiple reflective surfaces with both high accuracy and precision. Many varieties of SWI exist, often classified according to their experimental applications. Swept-source optical coherence tomography (SS-OCT), frequency-modulated continuous-wave (FMCW) lidar, and optical frequency domain reflectometry (OFDR) are among the more well-known [1]. Unlike SS-OCT, which is typically used for imaging biological tissue, FMCW lidar and OFDR are more often used to image targets which contain only a few reflective surfaces, often at relatively large standoff distances [2,3].

Coherent, heterodyne detection and large bandwidths provide SWI measurements with characteristically high signal to noise ratios (SNRs), in some cases 70dB or greater [1,2]. For target reflectors with sufficient spacing between them, these high SNRs allow measurements of reflector spacing to be super-localized, with resolutions well below the traditional Fourier transform limit.

In practice, temperature and pressure related fluctuations in the atmosphere, the target, and the SWI system itself are likely to set experimental limits on the accuracy and precision of any SWI measurement system [4,5]. In order to examine the fundamental limits of super-localized SWI, both in accuracy and precision, we take a numerical, rather than experimental approach. We generate data numerically, according to the specifics of a model SWI system, process it in the same way experimental data would be processed, and compare the results to the truth of the original model.

OVERVIEW OF NUMERICAL EXPERIMENTS

Numerical data was generated according to the specifics of the system shown in Fig. 1. This model is very similar to the experimental system described in [6]. In the measurement arm of the interferometer, two reflectors are located approximately 7% of the way through the unambiguous range of the system. The tunable laser sweeps from 1500nm to 1565nm, with a resulting sweep bandwidth of 8.3THz. A trigger interferometer with a 30m path length mismatch between the two arms sets sample spacing in optical frequency, as well as the total number of samples per laser sweep (830,096). The laser's tuning rate is assumed to be slow enough that second and higher order sweep rate nonlinearities can be neglected [7].

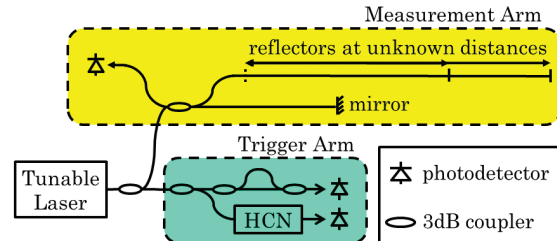


FIGURE 1. System diagram of a fiber-based SWI used as the model for numerical experiments. The Michelson geometry of the measurement arm enables the measurement of two (or more) reflective surfaces. The trigger arm consists of a Mach-Zender interferometer and a hydrogen cyanide reference cell.

Mathematically, the numerical data corresponding to a single laser sweep takes the form of Eq. 1, where $\nu[t]$ is optical frequency at sample times t , and the delay time τ_i to each reflector corresponds to a path length mismatch between arms of the measurement interferometer. Physical path length mismatch is obtained by multiplying delay time by the speed of light.

Zero-mean Gaussian noise $Z[t]$ is added to the signal by drawing values from a normal distribution whose variance is equal to noise power. In Fig. 2(a), a portion of this signal is plotted against laser wavelength λ .

$$U(v[t]) = 1 + \cos(2\pi v[t]\tau_1) + \cos(2\pi v[t]\tau_2) + Z[t] \quad (1)$$

The discrete Fourier transform (DFT) of the sampled signal contains peaks centered at delay times τ_i , given by

$$\tau_i = (n_i + \delta n_i) s_\tau \quad (2)$$

where n_i is the index of the highest-amplitude point in the DFT of the signal and δn_i is the fractional position of the peak between points. DFT sample bin size s_τ is equal to the inverse of laser sweep bandwidth; this is the Fourier transform limited resolution of the system.

SUPER-LOCALIZATION ACCURACY

It is apparent from Eq. 2 that the accuracy limits of delay time measurements depend both on uncertainty in DFT bin size and on uncertainty in super-localization. In general, uncertainty in DFT bin size depends on the method used to maintain sample spacing in optical frequency. Because tunable laser sweep rates are generally not linear, samples acquired at regular time intervals will be unevenly spaced in optical frequency [7]. Evenly spaced samples can be obtained in several ways, by resampling [8], linearizing the laser sweep [9], or using a secondary interferometer to trigger sample acquisition. The system modeled here uses a trigger interferometer that is calibrated using a wavelength reference gas cell. Consequently, uncertainty in DFT bin size ultimately depends on the quality of the gas cell [6,10].

Super-localization is the process of estimating, for each delay peak, the location δn_i of that peak inside a DFT bin. The accuracy of super-localization is determined by the quality of these estimates. Their quality is, in turn, determined by the estimation method itself. To our knowledge, the method described in [6] was the first applied to SWI data to obtain super-localized measurements. (Here, we refer to this method as Local Linear Regression (LLR) to disambiguate the estimation process from the

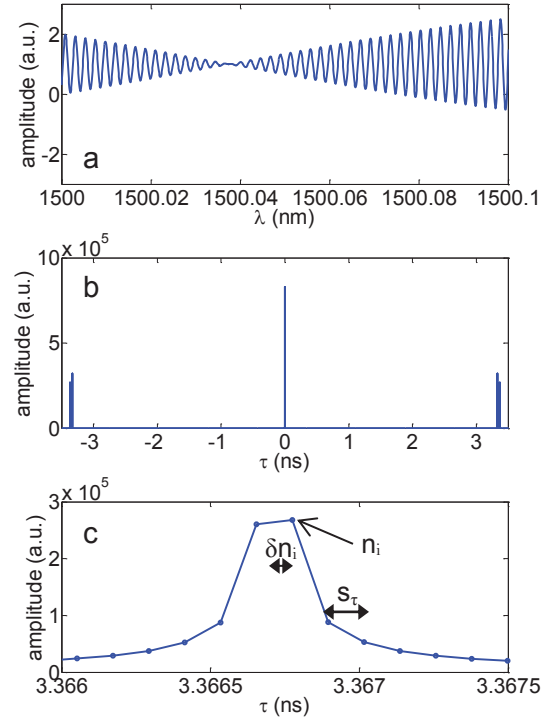


FIGURE 2. (a) Portion of measurement arm signal as a function of optical wavelength. (b) DFT of measurement signal contains a DC component as well as positive and negative sidebands with peaks corresponding to reflections from two surfaces. (c) Time delay to the second reflector is between sample points.

entire experimental process the authors of [6] call Precision Ranging.) However, many other methods of estimating the position of a peak between sample points can be applied to the same data. We take a well-known method from the frequency estimation literature, Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [11], and compare its performance against that of LLR.

ESPRIT

ESPRIT was originally developed to solve direction of arrival problems [12], but it is also used to estimate the frequency components of sampled signals [11], and here we use it to estimate exact delay times to reflectors. Because it is a parametric estimation method, the number of reflectors contributing to the signal data must be known advance. ESPRIT is an asymptotically unbiased and efficient estimator, generally considered to give good results [12]. Its speed depends on both the number of data points in the sampled signal and

on the order of the covariance matrix used in the estimation process.

Local Linear Regression

LLR is by far the faster of the two estimators (Table 1), largely because it is not a parametric method. Additionally, only a small portion of the signal is used to super-localize delay times of interest [6]. Delay peaks are windowed out, digitally filtered, and further processed to obtain super-localized estimates. However, these estimates are not unbiased. Consequently, because the uncertainty of LLR estimates of δn_i is non-zero, the use of LLR degrades the accuracy of delay time measurements.

TABLE 1. Run times for single estimates.

Estimator	LLR	ESPRIT
Run Time (s)	0.13 $\pm$ 0.01	26 $\pm$ 1

The bias of LLR estimates depends on several things: the locations of reflectors in the unambiguous range of the system, the location of delay peaks inside DFT bins, the number of points included in the windowing step, and the choice of digital filter. For the case of a single reflector, Fig. 3a illustrates how LLR bias

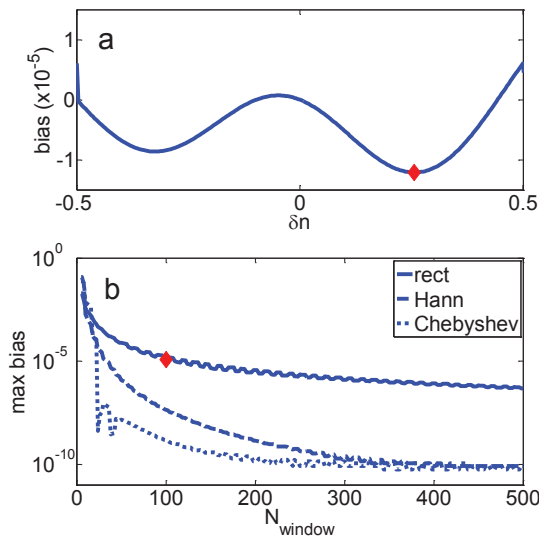


FIGURE 3. (a) Bias, normalized to DFT bin size, in estimated delay time for a single reflector. These LLR estimates were made using a 100 point rectangular window. (b) Maximum absolute bias, normalized to DFT bin size, of delay time estimates for a single reflector.

depends on the location of a delay peak inside a DFT bin. In general, bias decreases as the number of points included in the windowing step increases (Fig. 3b). Digital filter functions that decay smoothly to zero at their edges produce normalized bias values that asymptotically approach 10^{-10} . When reflectors are separated in delay time by approximately 200 DFT bins or more, LLR bias becomes inconsequential.

SUPER-LOCALIZATION PRECISION

To examine the variances of LLR and ESPRIT estimates, we simulate two scenarios based on the model in Fig. 1. The two reflectors in the measurement arm of the interferometer are the front and back surfaces of a fused silica window at 25°C, 5mm thick in one scenario, and 0.5mm thick in the other. At a series of SNRs, 510 simulations were run, 10 simulations each at 51 reflector locations spaced evenly across one DFT bin. Designated reflector separations were maintained at all reflector locations.

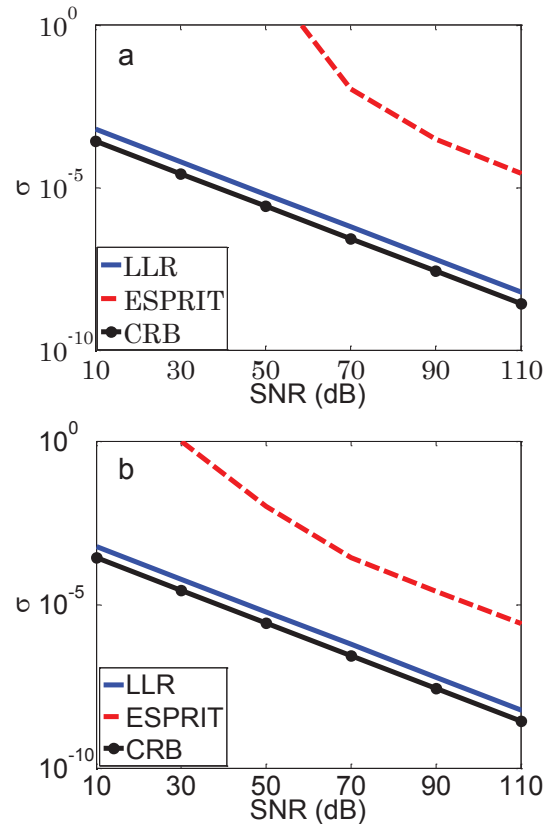


FIGURE 4. Normalized standard deviations of delay time estimates for reflectors separated by (a) 5mm and (b) 0.5mm of fused silica at 25°C. ESPRIT estimates were made using an order 100 covariance matrix.

In Fig. 4, standard deviations of recovered reflector positions are plotted, normalized to the size of a DFT bin. The square root of the Cramér-Rao Bound (CRB) is also plotted normalized to DFT bin size. For signals composed of reflections from multiple surfaces, there is typically no analytical solution for the CRB [13]. However, if the signal's parameters are known, the CRB may be estimated or, as was done here, calculated exactly using the method presented in [14].

As expected, the variance of LLR estimates increases slightly when reflectors are more closely spaced, since fewer points can be included in the LLR window. (For the 5mm spaced reflectors, 200 points were included in the LLR window, whereas for the 0.5mm spaced reflectors, only 40 points were included.) However, this increase is almost imperceptibly small, and certainly negligible in comparison to the differences in variance between LLR and ESPRIT estimates.

The variance of ESPRIT estimates increases quickly as reflector spacing decreases. A modified version of ESPRIT [15] can be used to increase the precision of ESPRIT estimates for closely spaced reflectors, but this comes at the cost of decreased unambiguous range for the

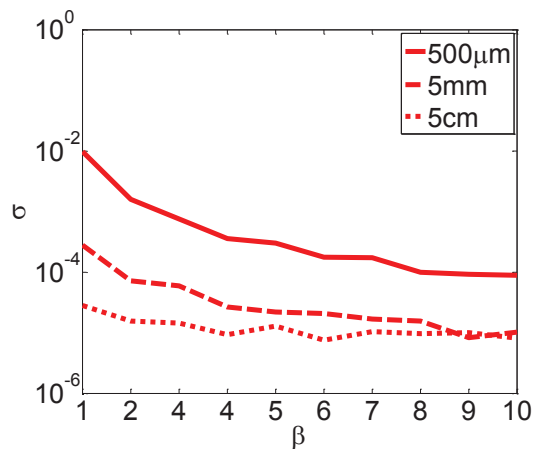


FIGURE 5. A modified form of ESPRIT artificially increases reflector spacing by downsampling signal data before making estimates. Here, normalized standard deviations of delay time estimates are plotted against the downsampling rate β . In these simulations, two reflectors were separated by three different thicknesses of fused silica at 25°C. SNR was 70dB. Each standard deviation was calculated from 30 simulations.

SWI system. Furthermore, the variances of these modified ESPRIT estimates are still higher than the variances of LLR estimates. This is shown in Fig. 5.

CONCLUSIONS

The ideal estimator for super-localized SWI measurements would be fast, unbiased, and noise tolerant. ESPRIT satisfies only one of these requirements. While ESPRIT estimates are unbiased, their noise tolerance suffers greatly when target reflectors are closely spaced. Additionally, for the system modeled here, which contains many points per data set and requires the computation of a large covariance matrix, ESPRIT is far slower than LLR.

By contrast, LLR is both fast and noise tolerant. It is not an unbiased estimator, but if target reflectors are sufficiently spaced—by at least 100-200 DFT bins in delay time—the bias of LLR estimates can be negligibly small. The utility of LLR demonstrates that the search for an ideal super-localization estimator need not be restricted to unbiased estimators. It also demonstrates how the best choice of estimator depends strongly on SWI system parameters and on experimental conditions. LLR and ESPRIT are only two among many multiple frequency estimators, and the well-developed field of frequency estimation likely has much to offer in the search for optimal estimators for super-localized SWI.

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SINUSOIDAL FREQUENCY MODULATION ON LASER DIODE FOR ITS FREQUENCY STABILIZATION AND DISPLACEMENT MEASURING INTERFEROMETRY

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ABSTRACT

In this paper, we propose the use of sinusoidal frequency modulation on a laser diode (LD) to achieve both frequency stabilization of the LD and a displacement measurement with a homodyne interferometer. The central frequency of the LD is stabilized to a Doppler broadened (linear) absorption or a Doppler free (saturated) absorption line of iodine (I_2) molecule near the wavelength of 633 nm. The frequency of the LD is modulated across the absorption line and the synchronous detection is utilized to find out the absorption signal. The homodyne displacement measuring interferometer can be constructed without high cost and complexity, and can attain high resolution in the measurement.

Keywords: modulation, frequency stabilization, laser diode, iodine, absorption, interferometer.

1. INTRODUCTION

A frequency modulated continuous wave (FMCW) interferometer, which was originated from radio detection and range (RADAR [1]), has many advantages. Theoretically, the FMCW interferometer can measure a long range which is limited by the coherent length of laser. Moreover, the small change of the range (= displacement: in this paper we define as this) can be detected [1 ~ 3]. Sinusoidal phase (frequency) modulation interferometer has been investigated for the displacement measurement [4,5]. Madden *et al.* have reported the availability of the sinusoidal frequency modulation on an LD to Michelson and grating interferometers [5]. They have used the Lissajous diagram to attain the displacement measurement using 2nd and 3rd harmonic modulated signals and the synchronous detections. One rotation (2π rad) in the Lissajous diagram coincides with the displacement of a half wavelength for a single path Michelson interferometer. If one degree ($\pi/180$ rad) can be resolved in the diagram, the resolution of 0.9 nm can be attained if the wavelength is 633 nm. However, to draw the exact Lissajous diagram with the sinusoidal frequency modulation, it is

needed to know the value of the modulation index [5]. In the sinusoidal frequency modulation for the Michelson interferometer, modulation index m varies with an angular modulation frequency ω_m , an angular frequency modulation excursion $\Delta\omega$, and a range L_0 (as we discuss in section 2 later). It means that the range L_0 can be determined from the modulation index m . The modulation index m can be calculated from harmonic intensities of an interference signal [6]. The combination method to use the Lissajous diagram and to determine the modulation index using the sinusoidal frequency modulation has been reported [7,8].

Recently, Makinouchi *et al.* proposed the use of the scanning probe position encoder (SPPE) decoding method for a sinusoidal frequency modulation grating interferometer [9]. The SPPE decoding method [10], basing on the combination of the phase-locked loop (PLL) and the null techniques, can measure a phase down to 10^{-5} rad. However, the method requires $J_2(m) = J_3(m)(J_i(m))$: Bessel function, i : an integer) or L_0 is fixed. However, the SPPE decoding method can be extended even though two consecutive Bessel function orders are not equal ($J_2(m) \neq J_3(m)$, or L_0 is variable) if the modulation index m is determined.

Sinusoidal frequency modulation technique is also effective to stabilize the frequency of an LD at a resonance frequency of molecules (I_2 , C_2H_2 , i.e.) [11~13]. If the frequency stability of the LD is improved, it is suitable for a light source of a high accuracy displacement measuring interferometer.

In this paper, we propose the combination method to stabilize the LD frequency to one of the I_2 molecule absorption lines, and to measure the displacement for a Michelson interferometer using the sinusoidal frequency modulation on the LD. First, we show the frequency stabilization of the LD to an I_2 linear absorption line and the displacement measurement using the Lissajous diagram method [8]. Secondly, we show the observation of the I_2 saturated absorption near 633 nm by use of an LD.

2. PRINCIPLE

2.1 Sinusoidal frequency modulation interferometer.

Figure 1 shows the sinusoidal frequency modulation Michelson interferometer. The interference signal on the photo detector is [2]

$$I(\tau, t) = I_0 \left\{ 1 + V \cos \left[\frac{\Delta\omega}{\omega_m} \sin \left(\frac{\omega_m \tau}{2} \right) \sin \omega_m t \right] \right. \\ \left. \left(t - \frac{1}{2} \tau \right) + \omega_0 \tau \right\} \quad (1),$$

where ω_0 , ω_m , $\Delta\omega$, τ , I_0 and V are a carrier angular frequency, the angular modulation frequency, the angular frequency modulation excursion, a delay time between two arms, an average intensity, a contrast of a beat signal, respectively [2].

In the interferometer, $\tau = \frac{2Ln}{c} = \frac{2(L_0 + \Delta L)n}{c}$, where L_0 , ΔL , n and c are the unbalanced length (range) between two arms, a displacement, the refractive index, and the speed of light, respectively. Actually $\frac{1}{2} \tau \ll 1$, Eq.(1) can be rewritten as the

$$I(L, t) = I_0 \left[1 + V \cos \left(m \sin \omega_m t + \omega_0 \frac{2Ln}{c} \right) \right] \quad (2),$$

where $m = \frac{\Delta\omega}{\omega_m} \sin \left(\frac{\omega_m \tau}{2} \right) = \frac{\Delta\omega}{\omega_m} \sin \left(\frac{Ln}{c} \omega_m \right)$ (3), is the modulation index.

Using the Bessel function, Eq. (2) is expanded as

$$I(L, t) = I_0 \left\{ 1 + V \left\{ \cos \left(\omega_0 \frac{2Ln}{c} \right) [J_0(m) + 2 \sum_{k=1}^{\infty} J_{2k}(m) \cos 2k\omega_m t] \right. \right. \\ \left. \left. - \sin \left(\omega_0 \frac{2Ln}{c} \right) 2 \sum_{k=1}^{\infty} J_{2k-1}(m) \cos(2k-1)\omega_m t \right\} \right\} \quad (4).$$

The unbalance length L can be assumed as the equation below

$$L = L_0 + \Delta L = N \cdot \frac{\lambda_0}{2} + \Delta L \quad (5),$$

where N is an integer, $|\Delta L| \leq \frac{\lambda_0}{2}$, λ_0 is the center wavelength of the laser. In Eq. (5) we assume L_0 equals products of N and the half wavelength. By combining Eq. (4), (5) and $\omega_0 = 2\pi \frac{c}{\lambda_0}$, we have

$$I(\Delta L, t) = I_0 \left\{ 1 + V \left\{ \cos \left(\frac{4\pi n}{\lambda_0} \Delta L \right) [J_0(m) + 2 \sum_{k=1}^{\infty} (J_{2k}(m) \cos 2k\omega_m t)] \right. \right. \\ \left. \left. - \sin \left(\frac{4\pi n}{\lambda_0} \Delta L \right) 2 \sum_{k=1}^{\infty} (J_{2k-1}(m) \cos(2k-1)\omega_m t) \right\} \right\} \quad (6).$$

In order to reduce the effect from the amplitude modulation of the LD, the synchronous detections of the 2nd and 3rd harmonics are selected. Using the lock-in amplifiers we can extract two signals, the 2nd and the 3rd harmonic terms, from Eq. (6) as below [5]

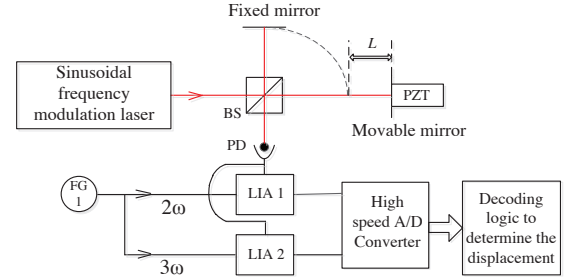


FIGURE 1. Sinusoidal frequency modulation Michelson interferometer; BS: beam splitter; PD: photo detector; LIA: lock-in amplifier; PZT: piezo electric actuator.

$$I_2 = I_0 V J_2(m) \cos \left(\frac{4\pi n}{\lambda_0} \Delta L \right) \quad (7),$$

$$I_3 = -I_0 V J_3(m) \sin \left(\frac{4\pi n}{\lambda_0} \Delta L \right) \quad (8).$$

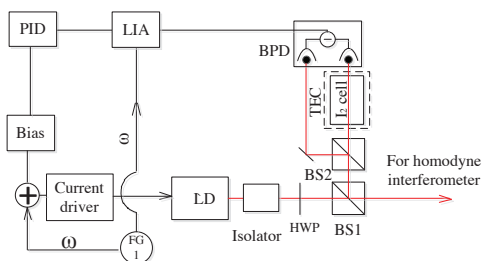
From Eq. (7) and Eq. (8) we can obtain the Lissajous diagram. The Lissajous diagram can be utilized for measuring the phase shift. The displacement ΔL can be calculated from the phase shift using following equation

$$\Delta L = \frac{\lambda_0}{4\pi n} \left\{ \arctan \left(-\frac{J_2(m)I_3}{J_3(m)I_2} \right) \right\} \quad (9).$$

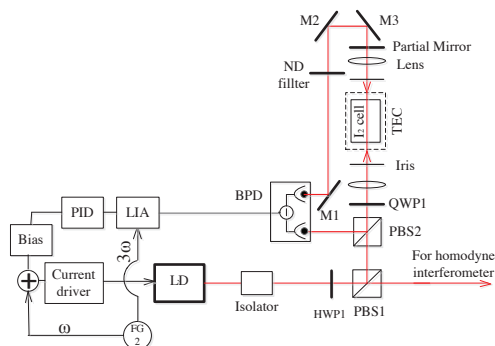
From the Eq. (9) we can determine the displacement using I_2 , I_3 , $J_2(m)$, and $J_3(m)$. To draw the exact Lissajous diagram, we must know the value of the modulation index m . From the power spectrum of the interference signal shown in Eq. (6), we can determine the modulation index m . If the real time determination of m is attained, the continuous (real-time) ΔL and L_0 measurements are possible from Eqs. (3) and (9). The SPPE technique is based on the equation as the same as Eq. (6) [9,10]. It means that we can apply the SPPE decoding logic to determine the displacement. The SPPE technique can achieve the phase resolution in 10^{-5} rad order. Previously, the SPPE method was applied for a grating interferometer which the range L_0 is fixed and $J_2(m) = J_3(m)$ [9]. However, we can apply the SPPE method for our interferometer even though L_0 is variable and $J_2(m) \neq J_3(m)$ if the modulation index m can be measured real-time.

2.2 Frequency stabilization of an LD to an I_2 absorption line

Figure 2a shows a configuration of the frequency stabilization of an LD to an I_2 linear absorption line. To detect the linear absorption lines of the I_2 molecule, the frequency of the LD is sawtooth-scanned by changing the injection current. A ba-



a. Frequency stabilization of a laser diode to an I_2 linear absorption line.



b. Frequency stabilization of a laser diode to an I_2 saturated absorption line.

FIGURE 2. Schematic diagram of the I_2 frequency stabilized laser diode; LD: laser diode; HWP: half wave plate; QWP: quarter wave plate; M: mirror; BS: beam splitter; PBS: polarized beam splitter; BPD: balanced photodetector; FG: function generator; TEC: thermoelectric cooler; LIA: lock-in amplifier.

anced photodetector (BPD) is utilized to detect the absorption signals and reduces the effect of the intensity fluctuation of the LD. The linewidth of these transition lines are broadened due to Doppler broadening effect. However, the frequency stabilization of the setup is expected to be $10^{-7} \sim 10^{-8}$ order [11]. We apply the sinusoidal frequency modulation technique to stabilize frequency of the LD to the center of the I_2 linear absorption line at near 633 nm (the P33 transition). The frequency of the LD is modulated by applying the sinusoidal modulation injection current, and the first harmonic signal of the absorption line is obtained using a lock-in amplifier (LIA). To stabilize the laser frequency, the output signal from the LIA is fed to current driver via a proportional-integral-derivative (PID) controller. The LD, whose frequency is stabilized to the linear absorption line of the I_2 molecule, is then used for the interferometer (see Fig. 1). A configuration of the I_2 frequency stabilized LD to

an I_2 saturated absorption line is shown in Fig. 2b. Two lens and two iris are employed to improve the overlapping between the probe and the saturated beams. We use a partial mirror (10 % of reflection) to ensure the power ratio of the probe and the saturated beams of 1:10. The frequency of the LD is modulated by sinusoidal modulation injection current, and the third harmonic signal of the absorption line is obtained using a lock-in amplifier. The frequency stabilization of the setup is expected to be 10^{-12} order [12].

3. EXPERIMENTAL RESULTS

3.1 Frequency stabilization of an LD to an I_2 linear absorption line and the displacement measurement.

In the experiment, we used a normal laser diode (HL63163DG Oclaro, Inc.) whose power was 30 mW at near 633 nm. This LD was fixed on a mount (Thorlabs, Inc.). The output laser light was directed through an 80 mm long I_2 cell (Ophos Instruments, Inc.). The temperature of the cold finger of the I_2 cell was stabilized at 15 ± 0.1 °C using a thermal electric cooler. A sawtooth frequency sweep with the frequency excursion of approximately 4.2 GHz and the period of 1s was applied to the LD to probe the absorption. The wavelength of the LD was concurrently measured using a wavelength meter (WS6, High finesse) with measurement resolution of 100 MHz. The first derivative signal was detected by a lock-in amplifier (in Fig. 2a, LIA; NF Corp.). A modulation frequency f_m of 3 MHz and a frequency modulation excursion Δf of approximately 410 MHz were used. The absorption line and the first derivative signals are shown in Fig. 3. In the figure, the bottom of the absorption line is at 632.992 nm (P33 6-3 transition). As show in Fig. 3, the null point of the first derivative signal coincides with the bottom of the absorption signal. It mean that the frequency of the LD can be stabilized at the central of the I_2 absorption transition using the null method. Then the first derivative signal can be fed to the PID controller to stabilize the LD using the null method. The comparison of the wavelength changes in both the free-run and the stabilized operations is shown in Fig.4 using the wavelength meter. Figure 5 shows a beat frequency measurement between the I_2 stabilized LD and an I_2 stabilized He-Ne laser (NEOARK Corp.). In Fig.5, the frequency fluctuation (peak to peak) was about 100 MHz ($\Delta f / f \approx 2 \times 10^{-7}$). The LD whose frequ-

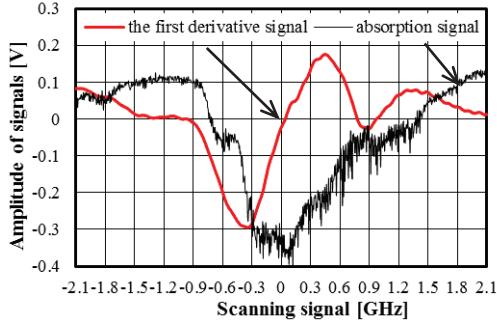


FIGURE 3. I_2 linear absorption detection

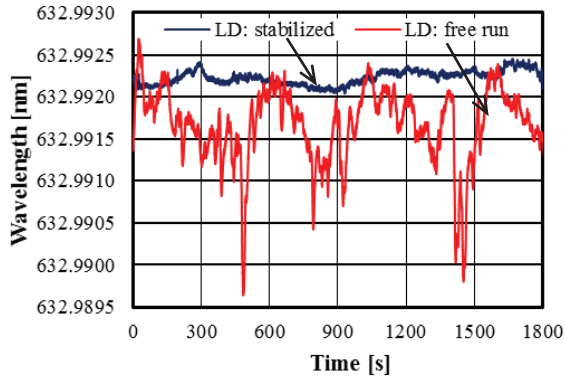


FIGURE 4. Wavelength comparison between the free-run LD and the stabilized LD in 1800 seconds.

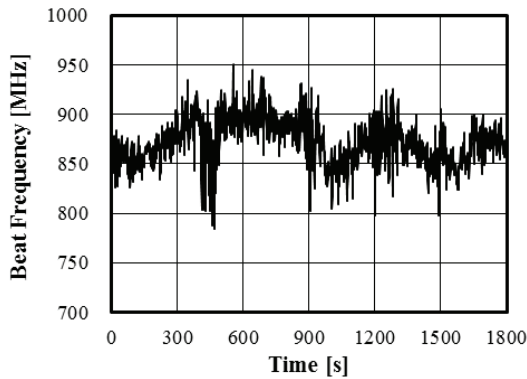


FIGURE 5. Beat frequency measurement between I_2 stabilized He-Ne laser and the stabilized LD.

ncy is stabilized to the I_2 linear absorption line at near 633 nm, is then used as the light source of the homodyne interferometer (see Fig. 1). We measured the displacements with the unbalanced lengths L_0 of approximately 0.2 m. Normally, the Lissajous diagrams are deformed due to the variation of the intensity of the laser source, and the other reasons. However, we can get the normalized Lissajous diagrams using fol-

-owing equations [5]

$$I'_2 = \frac{I_2 J_3(m)}{\sqrt{[I_2 J_3(m)]^2 + [I_3 J_2(m)]^2}} = \cos\left(\frac{4\pi n}{\lambda_0} \Delta L\right) \quad (10),$$

$$I'_3 = \frac{I_3 J_2(m)}{\sqrt{[I_2 J_3(m)]^2 + [I_3 J_2(m)]^2}} = \sin\left(\frac{4\pi n}{\lambda_0} \Delta L\right) \quad (11).$$

The center and radius of the normalized Lissajous diagrams are origin and 1, respectively. Sudarshanam *et. al.* [6] have proposed the following equation to determine m from the harmonic intensities

$$m^2 = \frac{4i(i+1)V_i V_{i+1}}{(V_i + V_{i+2})(V_{i-1} + V_{i+1})} \quad (12),$$

where $i > 1$, V_i is intensity of the i th harmonic component in Eq. (6). In the method, the sign of V_i must be positive, because a spectrum analyzer displays only the absolute magnitudes of spectral components. It means the measurable range of m is limited. We used the second to the fifth harmonics to calculate the modulation index m as following equation

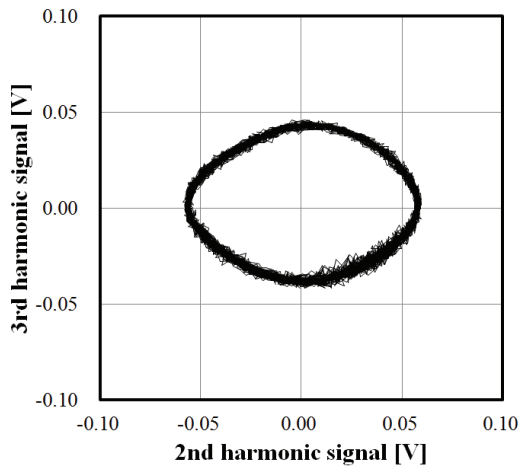
$$m^2 = \frac{48V_3 V_4}{(V_3 + V_5)(V_2 + V_4)} \quad (13).$$

In this case, the measurable range of m is less than approximately 5 rad. Figure 6 shows harmonic intensities $V_2 \sim V_5$ using a spectrum analyser (Anritsu Company) with the unbalanced length L_0 of approximately 0.2m. The modulation index m was determined to be approximately 2.7 rad using Eq. (13).

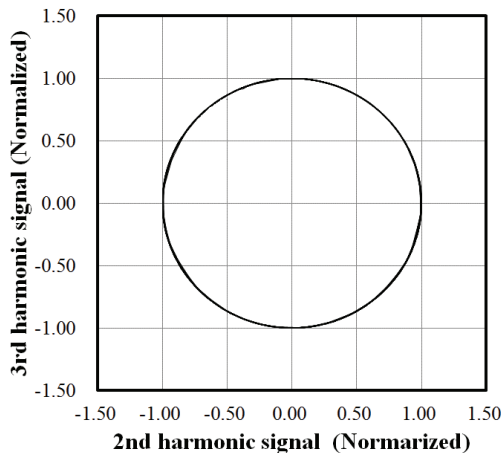
The moving mirror was driven by a piezo electric actuator (PZT, Physik Instrumente). We applied a triangular signal with an amplitude of 1 μ m and a frequency of 1 Hz to the PZT. Two simple lock-in amplifier circuits were made to detect the second and the third harmonics by combining mixers (R&K Co., Ltd.) and low pass filters (R&K Co., Ltd.). We used a modulation frequency f_m of 3 MHz and a frequency modulation excursion Δf of approximately 410 MHz in the experiment. The Lissajous diagrams from the 2nd and 3rd harmonics are shown in Fig.7. From the raw Lissajous diagram, we can make the normalized Lissajous diagram using Eqs. (10), (11). In Fig. 8, the displacement measurement using the LD in both the free-run (a) and the frequency stabilized (b) operations are shown. To see the displacement measurement error, the measured displacement was compared with an internal capacitive sensor integrated in the PZT actuator. Its measuring range and the resolution were 1 μ m and 1 nm, respectively. By using the frequency stabilized LD, we can improve the displacement measurement.



FIGURE 6. Harmonic intensities detection using spectrum analyser. Central frequency: 9.9 MHz; frequency span: 20 MHz.

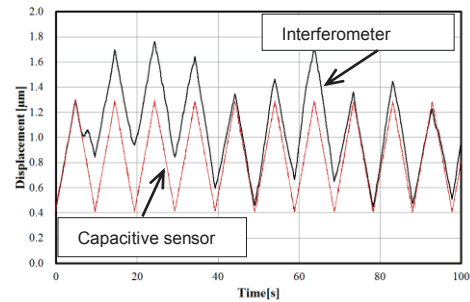


a. Raw Lissajous diagram.

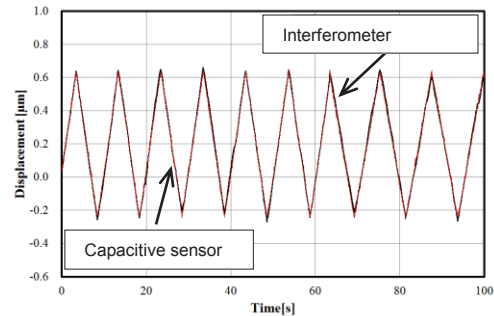


b. Normalized Lissajous diagram.

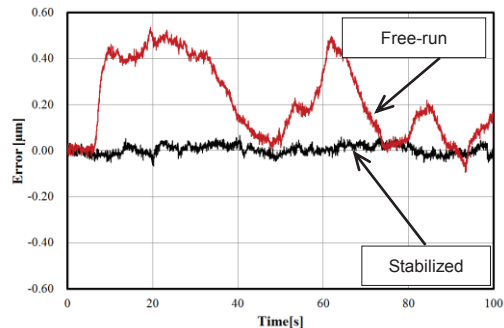
FIGURE 7. (a) Raw Lissajous diagram and (b) Normalized Lissajous diagram using the I_2 frequency stabilization of the LD.



a. Displacement measurement result using the LD in the free-run operation.



b. Displacement measurement result using the LD in the stabilized operation.



c. Comparison of the displacement error using the free-run LD and the stabilized LD operations for 1000 s.

FIGURE 8. The displacement measurement; unbalanced length: 0.2 m; modulation amplitude: 410 MHz; modulation index: 2.7 rad.

3.2 I_2 saturated absorption near 633nm.

An external cavity laser diode (ECLD, New Focus) was used to observe saturated absorption lines of I_2 near 633 nm. The central wavelength of the ECLD was near 633 nm and its linewidth was less than 300 kHz. The collinear setup was employed. The saturated and probe beam were 1.6 and 0.16 mW, respectively. 1 GHz frequency scan around P33 transition was made by grating actuation of the ECLD. A modulation frequency of 300 kHz and a frequency modulation excursion of

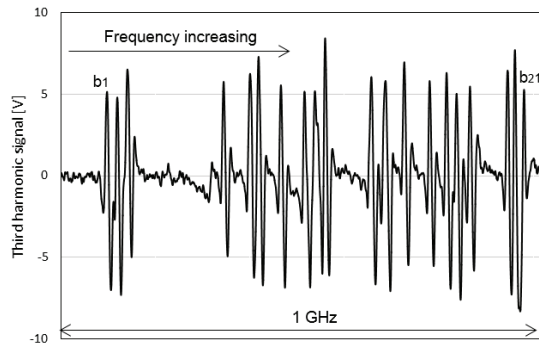


FIGURE 9. Hyperfine spectrum of P33 transition attained by third harmonic detection.

15 MHz were applied to ECLD by modulating the injection current. Figure 9 shows the saturated absorption components of P(33) transition of I_2 molecule by our instruments. The components b_1 - b_{21} obtained by the third harmonic detection can be seen clearly. The frequency stability measurement is not completed. However, the frequency stability is expected to be 10^{-12} order.

4. CONCLUSION AND NEXT WORK

(1). We performed the use sinusoidal frequency modulation ($f_m = 3$ MHz, $\Delta f = 410$ MHz) to stabilize frequency of the normal LD at the linear absorption near 633 nm and to measure the displacement using the calculated modulation index and the Lissajous diagram, simultaneously. The experiments confirm that our method can attain both frequency stabilization of the LD and high resolution displacement measurement.

(2). The I_2 molecule saturated absorption near 633 nm has been detected.

For the future, we will improve the frequency stability of the LD by locking to an I_2 saturated absorption component, and construct a displacement and range measuring interferometer using sinusoidal frequency modulation. We also have a plan to modify the SPPE decoding logic to get higher measurement resolution for the general Michelson interferometer even though the range L_0 is variable.

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Why Are We Using Step Height Standards to Calibrate Interferometers?

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In interference microscopy, the common practice is to calibrate the amplification coefficient by means of a material measure in the form of a physical artifact, most often a step height standard (SHS) such as chrome-coated etched glass [1]. The resulting uncertainty in the height scaling is 0.5 to 1%. This is disappointing, given that interferometry is the most common way of directly linking physical displacements to the meter via the wavelength of light. In a laser Fizeau interferometer, for example, it would be senseless to adjust such an instrument to match exactly the stated value for a mechanical SHS, as this would only degrade the uncertainty.

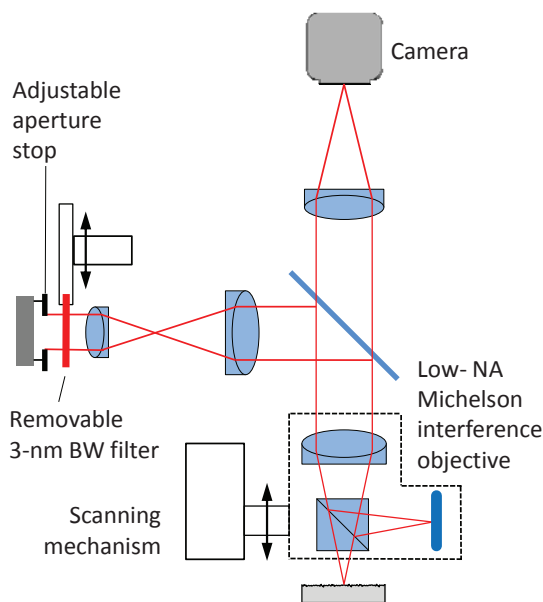


FIGURE 1. System layout for the proposed *in situ* calibration for the amplification coefficient of a coherence scanning interference microscope based on a calibrated, traceable central narrow-bandwidth illumination spectrum.

Evans and Davies have argued that in many cases, a calibration independent of a national metrology institute certification is not only appropriate, but can be more effective and easier to perform [2]. In this spirit of independence, we propose an *in situ* determination of the amplification coefficient (height scale) for an

interference microscope as an alternative to the traditional approach of using step height standards [3]. The method begins with a characterization of the spectral emission of the microscope illuminator temporarily equipped with a narrow-band spectral filter. The spectrometer employed for this characterization is calibrated using the 546.074nm ¹⁹⁸Hg line from a pencil lamp as a reference, providing direct traceability to the meter without the need for a certified physical artifact. Low magnification and a limited illumination aperture reduce geometrical effects such as the obliquity factor to acceptable uncertainties well below 0.05%.

Data acquired with the interference microscope links the wavelength standard to a calibration of the scan rate of the optical path length scanning mechanism of the interferometer. A capacitance gage embedded into the PZT type objective scanner assures stability between calibrations.

Results from single- and multiple-system testing for correlation and stability support a targeted k=2 uncertainty of 0.2%. This performance compares favorably to the more common calibration using step height standards, while being easier to perform and less sensitive to operator error.

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Interferometric Metrology for Precision Engineering - a Report From the Front Lines -

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Interference Microscopy (Coherence Scanning Interferometry [CSI] per the new ISO 25178 Areal Surface Texture standard) has long been used to characterize surface roughness for optical applications. As the CSI technology evolved and became more useful for “rough” surfaces characterizations, many segments of industrial manufacturing found application of this high speed data collection, 3D, production ready technology.

This presentation will highlight a number of applications relating CSI data to new ISO 25178 drawing callouts that define a functional characteristic of the component measured. One such application is the specification of “Scatter” as a functional control for an optical lens surface depending on rms as the functional control measurand.

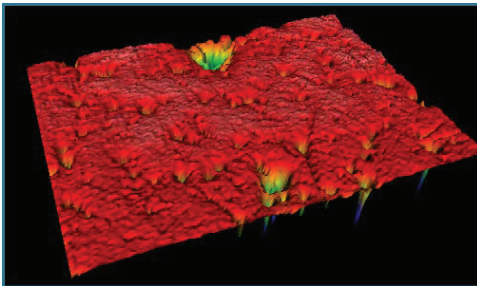


FIGURE 1. Polished lens surface of rms 4.30 nm

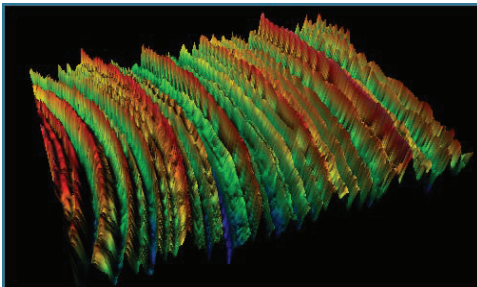


FIGURE 2. Diamond Turned lens surface of rms 4.10 nm

Figure 1. and 2. report lens surfaces generated by very different processes for the same application meeting the drawing specification having equivalent RMS values. The scatter

resulting from these surfaces will produce very different performance characteristics for these optics according to application.

Figures 3. and 4. demonstrate one of the new ISO parameters, “Str” (Texture Aspect Ratio - a measure of the spatial isotropy or directionality of the surface texture), that easily differentiates the surfaces and reports a result that can be used as a drawing callout.

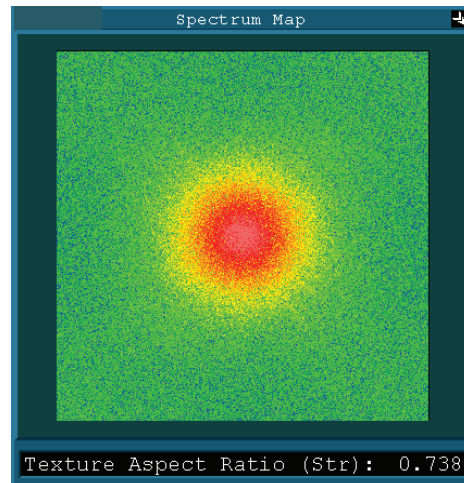


FIGURE 3. Polished lens surface is isotropic with an Str approaching 1.0

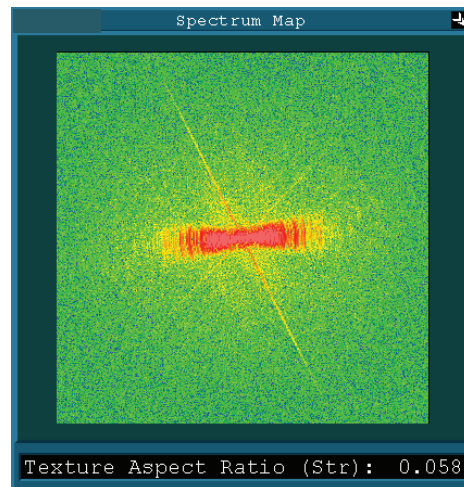


FIGURE 4. Diamond Turned lens surface is unidirectional with an Str approaching 0.0

IN-SITU DEFECT DETECTION SYSTEMS FOR R2R FLEXIBLE PV BARRIER FILMS

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INTRUCTION

The atomic layer deposition technique (ALD) is used to apply a thin (40-100 nm thick) barrier coating of Al₂O₃ on polymer substrates for flexible PV cells, to minimise and control the degradation caused by water vapour ingress. However, defects appearing on the film surfaces during the Al₂O₃ ALD growth have been seen to be highly significant in deterioration of the PV module efficiency and lifespan [1]. In order to improve the process yield and product efficiency, it is desirable to develop an inspection system that can detect transparent barrier film defects in the production line during film processing. Off-line detection of defects in transparent PV barrier films is difficult and time consuming. Consequently, implementing an accurate in-situ defects inspection system in the production environment is even more challenging, since the requirements on positioning, fast measurement, long term stability and robustness against environmental disturbance are demanding. For in-situ R2R defects inspection systems the following conditions need to be satisfied by the inspection tools. Firstly the measurement must be fast and have no physical contact with the inspected film surface. Secondly the measurement system must be robust against the environmental disturbance inspection. Finally the system should have sub-micrometre lateral resolution and nanometre vertical resolution in order to be able to distinguish defects on the film surface. Optical interferometry techniques have the potentially to be used as a solution for such application. However they are extremely sensitive to environmental noise such as mechanical vibration, air turbulence and temperature drift. George [2] reported that a single shot interferometry system "FlexCam" developed by 4D Technology being used currently to detect defects for PV barrier films manufactured by R2R

technology. It is robust against environmental disturbances; but it has a limited vertical range, which is restricted by the phase ambiguity of the phase shift interferometry. This vertical measurement range (a few hundreds nanometres) is far less than the normal vertical range of defects (a few micrometres up to a few tens micrometres). It is not possible to detect the majority of defects in the R2R flexible PV barrier films.

In recent year two interferometric technologies, Wavelength Scanning Interferometry (WSI) and White Light Channelled Spectral Interferometry (WLCSI), have been studied for in-situ surface inspection at the EPSRC Centre for Innovative Manufacturing in Advanced Metrology, University of Huddersfield [3-5]. The WSI system developed at our Centre combines an active environmental disturbance compensation system and GPGPU fast data processing technologies which enable the WSI system to be used at the shop floor for precision surface inspection. The WLCSI system measures a surface profile in a single shot which makes it free from the requirements on the positioning and stability when the measurements are carried out on the shop floor and therefore has the potential to be used for in-situ surface inspection.

This paper reports on the development, deployment and integration of two in-situ PV barrier film defect detection systems; one of them based on WSI and the other based on WLCSI; into an R2R film processing line at the Centre for Process Innovation (CPI). Both systems are robust against environmental disturbances. They both have nanometre vertical resolution and micrometre lateral resolution, and up to 100 µm vertical measurement range. These systems have been tested and characterised, and initial results

compared to results from using laboratory-based instrumentation are presented.

WSI PRINCIPLE

The measurement principle of WSI is based on measuring the phase shift of a reflected optical signal using wavelength scanning techniques by using an acousto-optic tuneable filter (AOTF). The measurement system is composed of two interferometers that share a common optical path. The measurement interferometer illuminated by a white light source is used to acquire the three dimensional surface profile of the sample in real time. The reference interferometer illuminated by a superluminescent light emitting diode (SLED) is used to monitor and compensate for the environmental noise. As the two interferometers suffer similar environmental noise, the measurement interferometer will be capable of measuring surface information once the reference interferometer is "locked" into compensation mode. Light reflected by the sample and the reference mirror are combined by the beamsplitter to generate an interferogram. A Dichroic beamsplitter is used to separate the measured interferogram signal and the reference signal. The interferograms are detected by a high speed CCD camera. The selected light wavelength of an AOTF is determined by:

$$\lambda = \Delta n \alpha \frac{v_a}{f_a} \quad (1)$$

where Δn is the birefringence of the crystal used as the diffraction material, α is a complex parameter depending on the design of the AOTF, and v_a and f_a are the velocity and frequency of the acoustic wave respectively. The wavelength of the light that is selected by this diffraction can therefore be varied simply by changing the driving frequency f_a . As a result, different wavelengths of light will pass through the AOTF in sequence so that a series of interferograms of different wavelengths will be detected by the CCD.

Intensities detected by pixel (x, y) of the CCD camera that correspond to one point on the test surface, can be expressed by

$$I(x, y; k) = a(x, y; k) + b(x, y; k) \cos(2\pi k h(x, y)) \quad (2)$$

The optical path different is given by

$$h(x, y) = \frac{\Delta\phi(x, y, \Delta k)}{2\pi\Delta k} \quad (3)$$

Since the change of ϕ can be calibrated first by using an optical spectral analyser, the main issue here is how to calculate the phase change.

The schematic diagram of WSI is shown in Figure 1. It is employed to measure the surface topography of the barrier coating and is capable of generating surface maps with unambiguous height, without the 2π phase ambiguity limitation. The interferograms are produced with no mechanical movement but by scanning the wavelength of a halogen light in the visible region (683.4 nm-590.9 nm) using an acousto-optic tuneable filter (AOTF). Such a measurement methodology can provide significant enhancements in speed compared to comparable methods such as white light scanning interferometry (WLSI). In addition, the WSI can be stabilised against environmental disturbances by using an active control of the reference arm, thus enabling nanometre scale measurements with large amounts of environmental isolation [3, 4]. This active control consists of a reference interferometer, which provides positional feedback, and a piezo-electric transducer (PZT), which moves the reference mirror. The PZT is driven by a PI controller to track the altering in the optical path due to environmental disturbance such as mechanical vibration and refractive index drift.

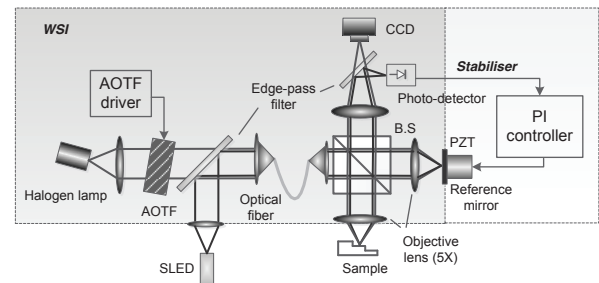


FIGURE 1. Configuration of the WSI

The reference interferometer is multiplexed with the WSI to share the same measurement optical path using an edge-pass filter (dichroic mirror) and a super luminescent diode (SLED) light source having a central wavelength of 820 nm. The intensity of the generated fringes is detected by a photo-detector which monitors the SLED light only, using a dichroic filter that reflects the IR light to the detector and allows the scanned visible light to pass through to the CCD. This control loop system can compensate for disturbances in the optical path length up to a few microns at 102 Hz and stabilise the WSI for wavelength scanning process.

During the measurement process, 256 interferograms are captured over a full field of view using 5X magnification objective lens and 640x480 CCD pixels. A periodic spectral interference pattern produced for each captured pixel is analysed individually using Fourier transform algorithm. This analysis is widely used to extract the phase from periodic fringe patterns by filtering out the DC interference term and the phase conjugate from the power spectral density obtained via the discrete Fourier transform. The extracted phase distribution suffers 2π ambiguity that can be removed using one-dimensional unwrapping method.

The evaluating process for areal topography requires long processing time when CPUs traditional sequential execution programs are used. The full analysis of all the pixels is, therefore, accelerated by parallelising the computation with a many-core graphic processing unit (GPU). The WSI can capture and generate a full areal topography in 3.7 seconds. The CUDA C program is used to achieve the data parallelism using a GPU device, hence increasing the measurement throughput, demonstrated. Typically, in a sequential programming model, the program generates a main thread that executes functions in a sequential manner. In contrast, the CUDA parallel programming model generates thousands to millions number of thread (typically equal to the captured frames size) to execute data-parallel functions, known as kernels, in a parallel manner. In this application, 307200 threads are generated to analyse each pixel individually in a parallel manner. The computing time is accelerated to approximately 1.1 second compared to 31.9 second using conventional sequential programming approach [6]. The atomic layer deposition technique (ALD) is used to apply a thin (40-100 nm thick) barrier coating of Al_2O_3 on poly

WLCSI PRINCIPLE

Different from WSI using an AOTF for wavelength scanning, WLCSI employs a diffraction grating to separate a series of constituent monochromatic interferograms which encode the phase as a function of wavenumber along the horizontal axis of the CCD camera (chromaticity axis) [7]. For a diffraction grating with space d , if a plane wave is incident with an angle θ_i , the diffraction angle is θ_m at the order m , then for a beam with wavelength λ , the following equation should be satisfied

$$m\lambda = d(\sin \theta_i + \sin \theta_m) \quad (4)$$

By selecting a proper grating spacing, the distance and the angle between the grating and CCD camera, it is expected that a desired spectral band of light can be captured with respect to the CCD pixels.

The basic configuration of the WLCSI in-situ surface inspection system is illustrated in Figure 2. A halogen bulb with broadband spectrum provides the white light illumination for the system, which is coupled into a multi-mode optical fibre patch cable with a numerical aperture of 0.39. The tested surface is observed through a cylindrical lens based Michelson interferometric objective. The measurement of long profiles can be achieved due to the imaging property of the cylindrical lens. The interference beam passes through a slit to block the light that is redundant for measurement. That is to say, for each measurement only a narrow line of light which represents an interference signal of a surface profile is selected, then diffracted by the grating and finally received on a CCD camera. After wavelength calibration wavenumber σ spreads along the chromaticity axis in a range of $1.50 \mu\text{m}^{-1}$ $1.71 \mu\text{m}^{-1}$. The direction of the slit is set to be parallel to the columns of CCD pixels, so that the dispersion axis is along the rows. The CCD camera (ICL-B0620 from Imperx) has a resolution of 648 x 488 pixels and a frame rate of 208 fps in normal working condition.

When the measurement is performed, the sample should be placed within the depth of field (DOF) of the objective to resolve the details of the tested surface. Fringes still can be observed as the optical path difference (OPD) between the tested surface and reference surface exceeds the DOF, the visibility of the fringes and the signal-to-noise of the interference output, however, greatly decreased and eventually no interference exist when the OPD is greater than the coherence length. Therefore, the measurement range is related directly to the DOF and coherence length, and determined by the smaller of these two numbers. The depth of field is defined by:

$$DOF = \lambda(1 - NA^2)^{1/2} NA^{-2} \quad (5)$$

The coherence length is expressed as

$$l_c = \frac{k\lambda^2}{\Delta\lambda} \quad (6)$$

where k is the correction factor depending on spectral profile. For the Gaussian distribution

and Lorentzian distribution, k is equal to 0.32 and 0.66, respectively. As for the WLCSI, the coherence length is much greater than the DOF due to the function of the spectrometer. Therefore, the measurable heights range is limited by the DOF, which is theoretically approximately 144 μm .

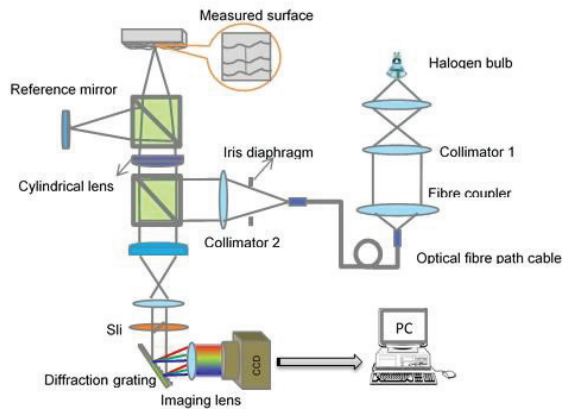


FIGURE 2. Schematic diagram of WLCSI

SYSTEMS IMPLEMENTATION

The WSI system has been mounted into a metrology frame using kinematic adjuster as described in Figure 3.a, resulting minimum tilt fringes across WSI field of view (i.e. less than 5 fringes per FOV). A porous air-bearing conveyor is placed underneath the polymer film to stabilise the web at fixed height. A standard artefact supplied by NPL has been mounted beside the edge of the foil web and aligned with respect to the conveyor in order to justify the WSI performance. The final setup of the system is shown in Figure 3.b.

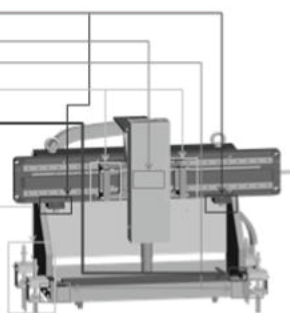
Alignment procedure

- Alignment at IBSPE

- I
- II
- III
- IV
- V

- Alignment at CPI

- VI



(a)



(b)

FIGURE 3. (a) Opto-mechanical inspection system, where I is align Y-stage w.r.t. conveyor in Z and R_x , II is align WSI w.r.t. Y-stage in Z, III align conveyor w.r.t. WSI in R_y , IV align WSI w.r.t. conveyor in R_x , V align artefact w.r.t. WSI in Z, R_x and R_y , VI align setup w.r.t. foil (b) Integration the inspection system onto rewinder stand.

The porous air-bearing conveyor is used for handling the film web which supports the film at fixed height and maximize the flatness of the web relative to the measurement plane. The handling solution should also be approved for use in clean room environment. The porous air-bearing conveyor can satisfy such conditions if uniform dry air pressure is supplied across the entire air bearing area with a vacuum pre-tension force to hold the foil. In this application, a conveyor type H-Series from New Way Ltd was used to supply uniform clean dry air (CDA) pressure, through a porous medium with a thin air gap, resulting in a more consistent fly height and better flatness across the full width of the flexible foil web. This technology can also act as a stop-gap filter, trapping any particles which may have escaped the filter system.

The conveyor performance was investigated by IBS Precision Engineering [8] using an optical sensor which scanned across the foil web after supplying 2 bar air pressure and vacuum pressure ranges 0-0.3 bar with 0.05 incremental step, see Figure 4. It has been found that the conveyor can hold the web with local height variation of $<5 \mu\text{m}$ under 0.05 bar vacuum. The worst case peak to peak variation is found to be $<25 \mu\text{m}$ over the 60 mm foil width without vacuum pressure. This height variation is within the focal depth limit of the objective lens used in WSI (e.g. the focal depth for 5X objective is equal to $\pm 14 \mu\text{m}$).

An effective auto-focusing methodology is crucial to the successful implementation of the WSI for large flexible substrate measurement. Although

the flexible substrate is held by a porous air-bearing conveyor, the surface undulation is still substantial across the 500 mm web width. Fast, automated positioning of the WSI head so its focal point is at the top layer of Al₂O₃ barrier is needed which must also be robust against the possibility of mis-focusing due to the multilayer structure of the web.

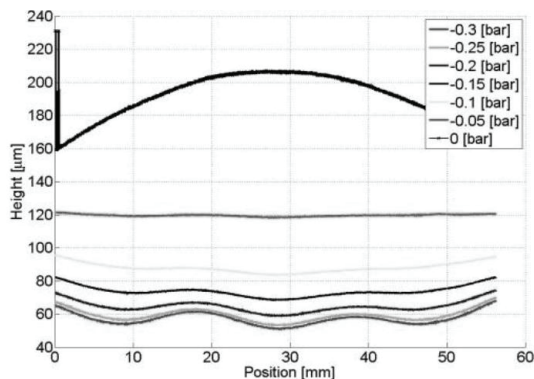


FIGURE 4. Air bearing performance at 2 bar air pressure when altering the vacuum pressure [8]

The auto-focus method is based on tracking the peak of the coherence envelope of the reference interferometer sourced by the SLED when the head is moved normal to the web to scan the focal plane of the WSI objective lens using a stepper motor. Simultaneously the intensity response is monitored by the reference interferometer, with the maximum intensity (coherence envelope peak) being found to be the point of focus. Throughout the translation, the position of the film is acquired continuously from the encoder of the stepper motor in order to map the physical position of the translation stage to peak position of the coherence envelope. This information allows the stage to move to the precise point of focus after the scan. The speed of the auto-focusing method is dependent on the speed of motorised translation stage. For the current setup, the auto-focus routine, using a 1 mm translation distance, took approximately 0.6 second.

The limiting factor for the auto focus method is the number of tilt fringes which results in the degradation of signal-to-noise ratio of the feedback signal. Experimentally, it has been determined that a sufficient interference signal can be obtained for effective autofocus operation when there is more than 5 fringes (equivalent to 2.4 mrad surface gradient) appear across the WSI field of view, see Figure 5.

Increasing the number of fringes across the active photo-detector area, which is equal to 0.5 mm², will average out the coherence intensity and hence degrade the feedback signal.

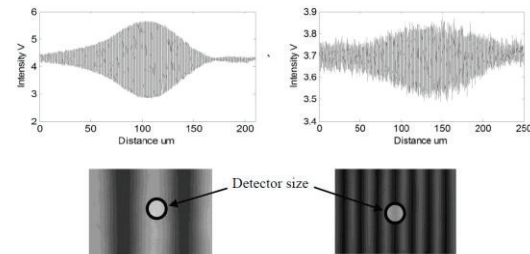


FIGURE 5. The coherence intensity response with its corresponding FOV.

The start position is located at the artefact such that the operation of the system is initiated by measuring the 1.2 μm step height sample. It has been found that the measurement precision is better than 20 nm. The WSI is translated incrementally over the web width while the foil is stationary. Instead of inspecting a foil coated by Al₂O₃ barrier, gold coated PET film has been measured to obtain preliminary such that three defects were detected as shown in Figure 6. The measurement throughput to cover 500 mm width is approximately 2 hours using 5X objective lens. The overall data size for full width (i.e. 500 mm x 0.7 mm) is larger than 300 megabyte.

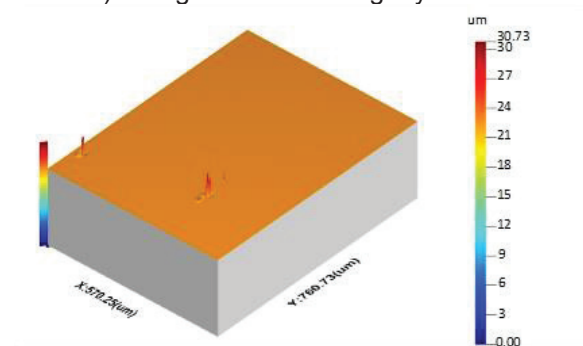


FIGURE 6. PET film areal measurement using WSI

The WLCSI system can be integrated into the metrology frame the same as the WSI system. Compare to the WSI system it does not have an auto-focus system which results the demanding for the metrology frame is high. With the specifications of the metrology frame and the air bearing system and the relative large vertical measurement range the prototype can be fitted into the R2R system for the inspection. A

measurement on a scratched glass surface is shown in Figure 7.

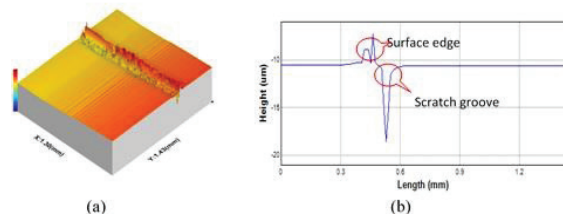


FIGURE 7. Measurement results of a scratch by WLCS: (a) 3D surface map, (b) cross-sectional profile.

CONCLUSION

It is well established that the efficiency of flexible PV is correlated to the WVTR which is dominated by the presence of large defects in the barrier layers. Implementation on-line defect detection systems can enhance the roll-to-roll (R2R) manufacturing process. The WSI and WLCSI systems can be considered as a solution for R2R process as combined to traverse and autofocus stages and air bearing conveyor. These two inspection systems can measure wide foil area in spite of environmental disturbances and without interaction by the operator for alignment or focusing the instrument.

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EXTENDING THE VERTICAL RANGE OF WAVELENGTH SCANNING INTERFEROMETRY

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INTRODUCTION

The mass production of high-precision components has created a demand for in-line inspection tools that are able to perform in-situ surface topography measurement as part of a quality assurance tool and as feedback to control the manufacturing process. Wavelength scanning interferometry (WSI) is a candidate for in-line inspection of manufactured parts with nanometre height features [1] [2] [3]. WSI has advantages over other widely known optical interferometric techniques, such as coherence scanning interferometry and phase shifting interferometry, as it involves no mechanical scanning, allowing for higher measurement speed, and does not suffer from 2π phase ambiguity. In this paper, a number of different improvements to the standard WSI signal processing technique are proposed that can extend the measurement range while increasing accuracy and resolution, with little to no impact on the measurement speed. In next section the WSI technique is described and an optimisation of the Fourier Transform Method (FTM) for phase slope estimation reported. The concept of the quadrature WSI (Q-WSI) is then described, and a simulation of the FTM algorithm and Q-WSI is compared and discussed.

WAVELENGTH SCANNING INTERFEROMETRY

In the WSI technique a phase shift in the recorded interference pattern is introduced by changing the wavelength of the interfering light. Figure 1 shows the setup for the WSI instrument [3]. The light source is a halogen lamp, with the light being spatially filtered by a pin hole and then collimated. The collimated beam passes through an acousto-optic tunable filter (AOTF), which dynamically filters the broadband light. By selecting the vibrating frequency of the AOTF crystal it is possible to

deflect only a narrow-band spectral line (FWHM ≈ 2 nm) which is then coupled into a fibre and delivered to the interferometer. The light is then collimated and goes through a Linnik interferometer and finally the fringe pattern intensity is recorded by a CCD camera. In the measurement process, a narrow-band spectral line is selected and an interferogram of the whole surface is recorded; this process is repeated for each selected wavelength in the desired range. Once the stack of images are recorded, they are processed in parallel with a graphic processing unit (GPU); the whole measurement and computation take less than 2 s, and is currently limited by the CCD camera frame rate. The interference signal for the pixel with coordinates (x, y) in the instrument field of view (FOV) is:

$$I_{xy}(k) = q[1 + V \cos(4\pi k h_{xy})] \quad (1)$$

where q is the average intensity, V is the fringe visibility, k is the wave wavenumber (the reciprocal of the wavelength) and h_{xy} is the surface distance from the zero optical path difference position. The interference signal is a function of the wavenumber (k), and in the measurement process the wavenumber is varied linearly and the intensity recorded. The phase shift introduced is therefore proportional to the surface distance being measured. The aim of the FTM algorithm is to demodulate the phase from the fringe pattern and to estimate the total phase change, which is proportional to the frequency. Finally, the measured surface distance (h) is proportional to the phase change through the scanned wavenumber range and a constant:

$$h = \frac{1}{4\pi} \frac{\Delta\varphi}{\Delta k} \quad (2)$$

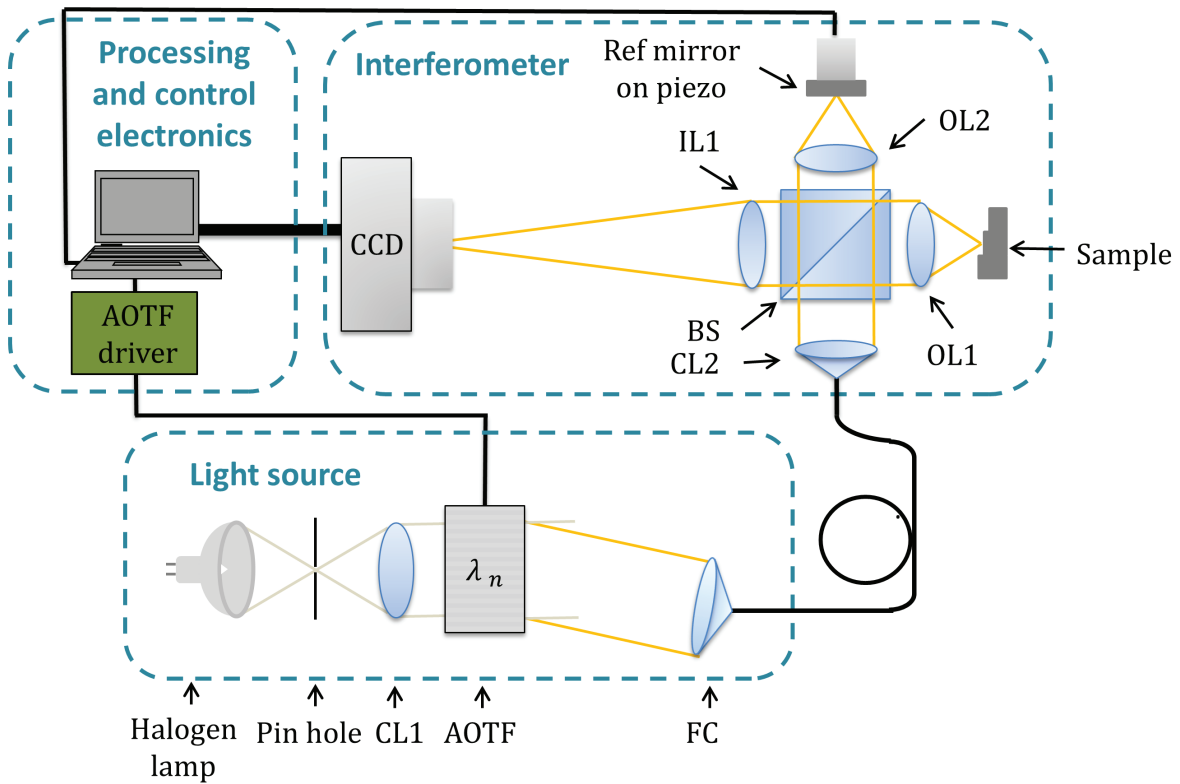


FIGURE 1. WSI setup: CL1, CL2: Collimation lenses 1 and 2; FC: fibre coupler; OL1, OL2: objective lenses 1 and 2; IL1: imaging lens 1; BS: beam splitter.

where $\Delta\varphi$ is the phase change and Δk is the scanned wavenumber range. The standard method employed to demodulate the phase is the Fourier Transform Method (FTM) [4]. The FTM consists of Fourier transforming the interference pattern and filtering the spectral peaks corresponding to the average intensity and the conjugate peak of the sinusoid. An inverse Fourier transform is then employed in order to extract the fringe pattern phase distribution. The phase distribution is linearly fitted in order to estimate the phase change and, therefore, the instantaneous frequency. However, for values of h corresponding to interference patterns with a non-integer number of periods, spectral leakage occurs which mixes the peaks in the spectral domain and, therefore, introduces an error in the phase demodulation and finally in the height estimation. The problem is similar to a piezo linear calibration error in phase shifting interferometry. The presence of a linear calibration error introduces sampling points which are not spaced by a phase of $\pi/2$, resulting in a non-integer number of fringe pattern periods. As a consequence,

the fringe pattern is not projected into a single bin of the Fourier transform leading to an error in the estimated phase. For phase shifting interferometry, the algorithm sensitivity to this error can be reduced by employing a data-sampling window [5][6]. The window's effect is to reduce the algorithm's sensitivity to higher order harmonics which appear in the presence of piezo calibration errors.

In WSI, the leakage is not a result of calibration error but it is intrinsic to the method. The fringe pattern frequency is not known *a priori*, but has to be estimated, and an ideal phase shift cannot be determined. Windowing the WSI fringe pattern reduces the spectral leakage, but does not eliminate it completely, and in some cases the window results in worse height error[7]. The height error is due to a distortion of the demodulated phase, which has a different shape depending on the window used and the amount of spectral leakage. To reduce the error in the height estimation it is possible to combine the windowing with a drop of the edge phase points before the linear fitting, in order to reduce the error in the height

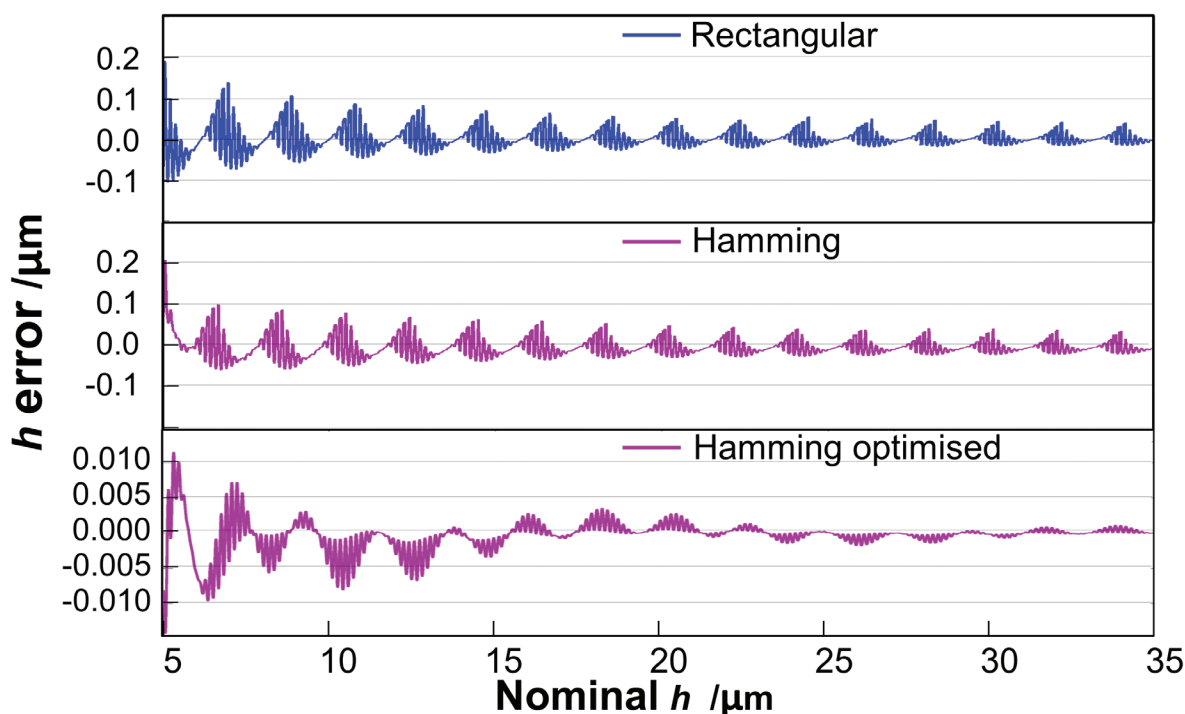


FIGURE 2. Comparison of height error estimated with the FTM for phase demodulation under different condition: (Top) No window applied; (Centre) Hamming window applied; (Bottom) Hamming window and phase edge drop points before linear fitting. Note the different y scale in the Hamming optimised case.

estimation. When a Hamming window is employed, the best results are obtained by dropping 12.5 % phase points on each side of the demodulated phase. Figure 2 shows a comparison of the height error for the non-optimised case (rectangular window), the simple windowed case (Hamming window), and the window with phase edge drop (Hamming optimised). The maximum height of 35 μm is selected as this corresponds to the depth of focus (DOF) of the microscope objective with 5x magnification and $\text{NA}=0.14$, currently employed in the WSI. For heights smaller than 5 μm , the FTM algorithm cannot demodulate the phase without large errors, since the spectral peak and its conjugate are too close to be separated, eventually merging in to a single peak for a constant fringe pattern corresponding to a height of zero. A detailed discussion of this optimisation is given elsewhere [7]. In the FTM, an important assumption is made by arbitrarily selecting the spectral peak corresponding to the positive or the negative frequency, *i.e.* fringe patterns with a higher frequency are considered closer or further away from the instrument depending on this choice. In fact, the value of h , estimated from the signal in equation 1, has a sign ambiguity due to the even

characteristic of the cosine. A signal corresponding to positive or negative values of h leads to the same exact recorded fringe pattern and an additional assumption has to be made. In order to correctly detect whether a point is a peak or a valley, the operator needs to know on which side of the zero OPD the surface is placed and select the peak or its conjugate accordingly. Therefore, the WSI working range is limited to only positive or negative heights with an absolute value larger than 5 μm (with respect to zero OPD).

QUADRATURE WSI

In order to solve the sign ambiguity, a possible solution is to record two interference patterns in quadrature, *i.e.* with a relative phase shift of a quarter of period. The complex interference pattern is obtained by arranging the two interference patterns on the real and imaginary axis (see figure 4). The complex interference pattern so obtained has a positive or negative frequency depending on the sign of the relative phase shift between the real and imaginary interference pattern. There are many algorithmic advantages of this method; firstly, it more than doubles the instrument working range. It allows positive and negative heights to be distinguished and

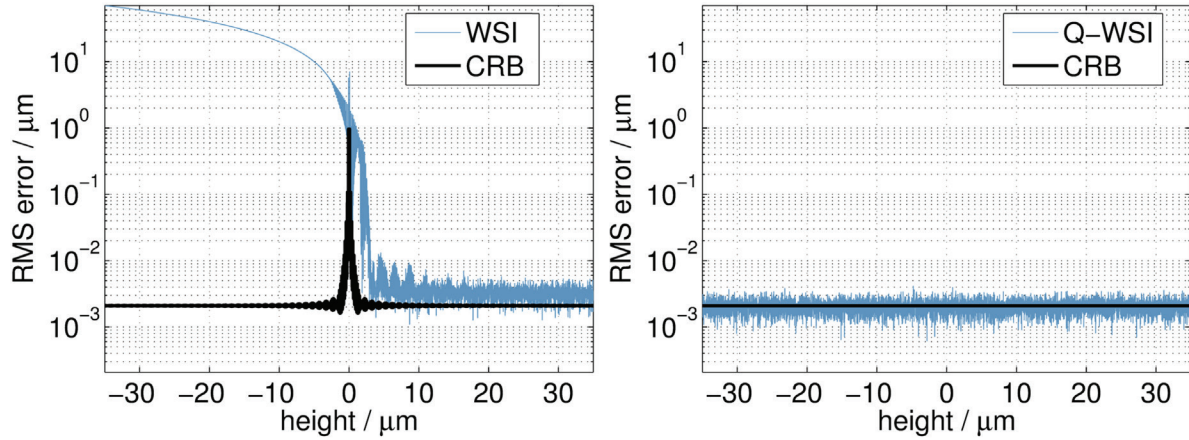


FIGURE 3. Simulation of height estimation for a real interference pattern process with optimised FTM algorithm (left) and angle from a complex interference pattern (right).

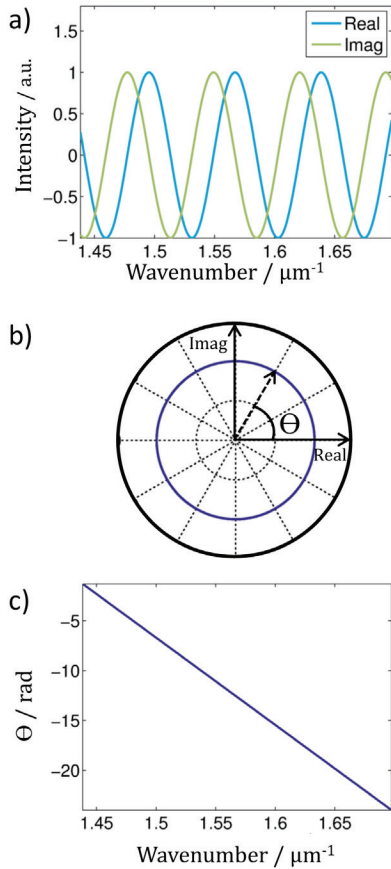


FIGURE 4. a: Simulated real and imaginary interference pattern for a height of approximately $7 \mu\text{m}$. b: relative Lissajous curve for the real and imaginary interference pattern in a. c: phase relative to the complex interference pattern.

the heights can be estimated between the range from $0 \mu\text{m}$ to $\pm 5 \mu\text{m}$. No spectral filtration or Fourier transform is needed to extrapolate the fringe pattern phase distribution, but the phase can be extracted directly from the angle of the complex fringe pattern and linearly fitted in order to estimate the phase slope. Figure 3 shows the root mean square (RMS) error in the height estimation for a real interference pattern processed with the Hamming optimised algorithm shown in Figure 2 and for the complex interference pattern. The RMS error is calculated by repeating the calculation with ten different Gaussian additive noise patterns and with a signal to noise ratio (SNR) of 30 dB. For both cases, 256 samples are acquired in total; for the real interference pattern, 256 samples are acquired over a wavelength range of 105 nm, for the complex case 128 samples are acquired for the real and the imaginary part of the interference pattern in order to compare the performance with the same number of total samples. In both cases the Nyquist-Shannon sampling theorem is respected. The performance of the algorithms are compared with the Cramer-Rao bound (CRB) [8][9], which establishes a lower bound on the frequency estimation due to the additive noise. The performance improvements are clear from Figure 3. In the Q-WSI case, positive and negative heights are distinguished easily. Furthermore, in the Q-WSI case, heights between $0 \mu\text{m}$ and $\pm 5 \mu\text{m}$ are correctly obtained. In Q-WSI, the increasing error when approaching the zero height is removed, and the ripple due to the spectral leakage is no longer present. Furthermore, the CRB of 2 nm is more closely ap-

proached by the Q-WSI. The importance of this new method is clear when considering a higher magnification objective lens. For an objective lens with an NA of 0.3 (10 $\times$), the DOF in the visible spectrum is approximately $\pm 7 \mu\text{m}$ ($\pm \frac{\lambda}{NA^2}$) and, therefore, a working range between 5 μm and 7 μm may be a limitation. The main drawback of the Q-WSI method is that some mechanical scanning is reintroduced since the phase shift is modulated by moving the reference mirror (see Figure 1). Other non-mechanical methods to record the quadrature signal may be possible, for example, the fast switching properties of a Pockels cell has been shown to allow fast phase shifting of a laser source [10]; however, for WSI, optical modulation of the phase shift for a broad range of wavelengths has to be addressed. The required phase shift steps do not depend on the surface heights as in CSI, or on the algorithm performance as in PSI, but on the sampling density in the scanned wavenumber range. Another possible source of error is the interference signal mean which needs to be known a-priori, in order to centre the complex signal around the origin. The measurement speed is not affected in the current setup since the CCD camera is the speed limitation of the system and the number of recorded frames is kept constant by distributing half on the real interference pattern and half on the imaginary component. The AOTF tuning time is smaller than 20 μs , therefore allowing a wavenumber scanning rate of 50 kfps, versus a camera frame rate of 200 fps. The piezo transducer settling time is smaller than 1 ms [11], therefore allowing a scanning rate of 1 kfps.

CONCLUSION

Two different improvements for a WSI instrument have been presented. The first improvement is an algorithm optimisation that allows a reduction in the height estimation bias in the FTM for phase slope estimation. The second improvement more than doubles the working range with respect to the previous case. Processing a complex interference pattern allows for no sign ambiguity, better estimation for heights corresponding to low frequency fringe patterns, completes removal of FTM non-linearity due to spectral leakage, and approaches more closely the CRB. However, the main drawback of the Q-WSI method is to reintroduce some mechanical scanning, adding the need for a piezo calibration step. Also, the interference signal background needs to be known a-priori, and therefore a calibration step for the

background is needed. Error in the piezo or background calibration may lead to error in the surface height determination. Future work will include implementation of the current algorithm and thorough study of error sources.

ACKNOWLEDGMENTS

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MEMS OPTOMECHANICAL ACCELEROMETRY STANDARDS

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INTRODUCTION

Primary accelerometer calibrations are often performed at National Metrology Institutes (NMI)[1], where the test units are mounted onto interferometrically-interrogated reference shaker systems[2, 3, 4], which typically reach relative uncertainties of the order of 10^{-2} – 10^{-3} . Reference accelerometers that have been calibrated at primary facilities, are used, in turn, to calibrate additional devices using back-to-back measurements. This latter approach, however, often results in reduced performance and higher uncertainty. In addition, there is an increasing need in industry, aerospace and defense applications for cost-effective in-the-field calibrations that minimize down time and drift of reference systems.

As part of the research funding initiative from the National Institute of Standards and Technology (NIST), NIST-on-a-Chip, we are developing portable chip-based accelerometers that incorporate displacement measurement interferometry to produce NMI-grade measurements that are directly traceable to the International System of Units (SI). The device design further detailed below, will allow both cost-effective production for wide deployment due its compatibility with MEMS semiconductor fabrication, and self-monitoring to ensure the specified measurement uncertainty over the certified lifetime of the device.

In addition to the improvements in acceleration metrology and calibrations, this research will greatly impact enabling technologies, particularly for inertial navigation systems. Current performance challenges, such as bias and scale factor stability, are tied to the uncertainty of the system, which will be directly addressed by this work. Similarly, research in seismology, ground and space-based geodesy, which base their observations in great extent to a traceable acceleration measurement, will benefit from such highly compact devices of significantly higher sensitivity and accuracy.

This article presents the sensor and measurement concept under study and preliminary results on the chip-based micro-optomechanical device fabrication and laser-interferometric results.

OPTOMECHANICAL ACCELEROMETER

Our concept for MEMS optomechanical acceleration standards is based on a micromechanical resonator that vibrates out of plane and incorporates a plano-concave Fabry-Pérot (FP) cavity to read out the motion of the accelerometer test mass, as shown in Figure 1. Furthermore, an additional stage can be added to the back side of the test mass for capacitive sensing and electrostatic force rebalance.

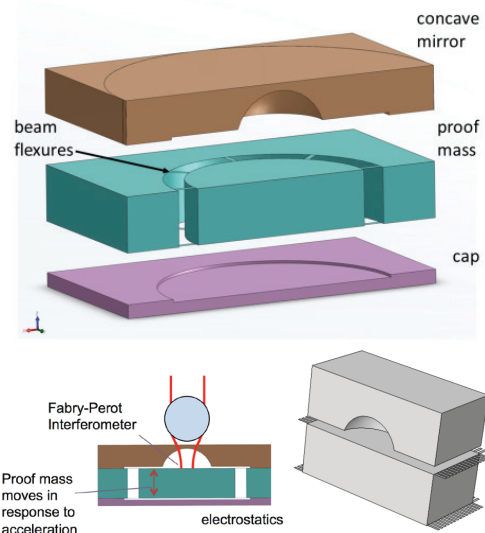


FIGURE 1. Schematics of the Si MEMS optomechanical accelerometer standard, incorporating a plano-concave Fabry-Pérot cavity.

Assuming a simple harmonic oscillator response for the mechanical resonator, acceleration can be obtained from a direct displacement measurement of the test mass by

$$\frac{X(\omega)}{A(\omega)} = -\frac{1}{\omega_o^2 - \omega^2 + i\frac{\omega_o}{Q}\omega}, \quad (1)$$

where $X(\omega)$ is the relative displacement given an

input acceleration $A(\omega)$ at an angular frequency ω , with a harmonic oscillator response of the mechanics determined by its natural frequency ω_o and quality factor Q .

Presently, we strive to achieve a relative measurement uncertainty below 10^{-3} in a dynamic range of 1 mg – 1 g over an observation bandwidth of 1 Hz – 20 kHz . The performance of accelerometers is commonly specified in units of g , which is defined as 9.80665 m/s^2 , referred to the SI. By integrating a laser interferometer into the system it is possible to measure the test mass displacement in terms of an optical frequency/wavelength standard.

In addition, as shown in Equation 1, a complete system identification consists of only two parameters, namely the resonance frequency ω_o and quality factor Q . Therefore, this knowledge is sufficient to compute acceleration from a displacement measurement with no additional information, assuming the accelerometer exhibits a linear response with a measurable damping mechanism. These parameters can also be measured and calibrated with respect to frequency standards, which provides traceability to the SI, and enables verification of device performance in the field.

DEVICE NANOFABRICATION

Hemispherical Mirrors

An outline of the nanofabrication process we have developed to fabricate the concave mirrors is shown in Figure 2. As a base substrate

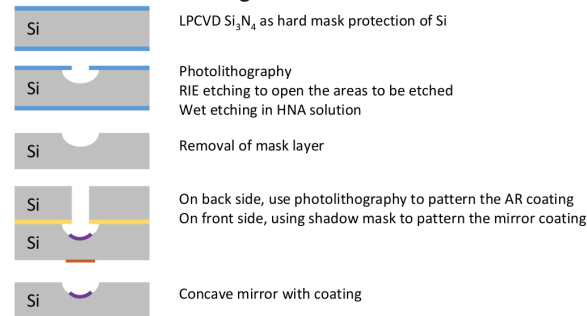


FIGURE 2. Outline of nanofabrication process of silicon hemispherical mirrors.

we utilize a silicon (Si) wafer with a thickness of $500\text{ }\mu\text{m}$, and deposit a 300 nm silicon nitride (Si_3N_4) layer as a hard mask on top of it by applying a low-pressure chemical vapor deposition (LPCVD). The wafer is then patterned lithograph-

ically using photoresist SPR 220.3, and this pattern is then transferred to the Si_3N_4 hard mask by reactive ion etching (RIE). The hemispherical concavity results from a wet etching process using a HNA solution of $\text{HF}:\text{HNO}_3:\text{CH}_3\text{COOH}$ with a ratio of 9:75:30 at room temperature. Subsequently, the Si_3N_4 hard mask is removed by phosphoric acid and the wafer is then sent for a final RCA cleaning. Once the mirror geometry is completed in the nanofabrication process, the optical properties of the final micro-optical components are defined by applying an anti-reflection coating to the back side of the wafer to reduce stray beams and undesired optical etaloning. Additionally, a typically high-reflectivity dielectric coating is applied to the front side of the wafer to define the input mirror of a high finesse optical cavity. Figure 3 shows scanning electron microscopy (SEM) images and post-processed white light interferometry (WLI) measurements, revealing a mirror geometry with a radius of curvature of typically $300\text{ }\mu\text{m}$, a depth of $220\text{ }\mu\text{m}$, an aperture diameter of $560\text{ }\mu\text{m}$ and surface roughness of approximately 1 nm_{rms} .

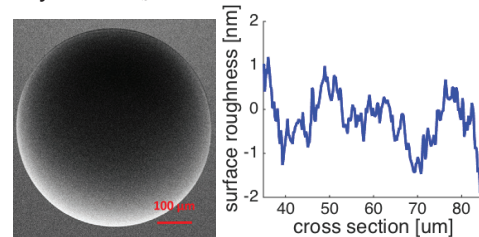


FIGURE 3. SEM and post-processed WLI measurements of hemispherical mirrors with a surface roughness of 1 nm_{rms} .

Mechanical Oscillator

The nanofabrication process developed for our mechanical resonators is outlined in Figure 4.

Similar to the case of the hemispherical mirrors, we use a $500\text{ }\mu\text{m}$ thick double-sided polished silicon wafer, and create a $3\text{ }\mu\text{m}$ deep spacer using HNA etching with a Si_3N_4 hard mask. The layer to produce the released-beam resonator pattern is achieved via Si_3N_4 LPCVD. Following the coating of the mirror using a lift-off process, a $1\text{ }\mu\text{m}$ low temperature oxide (LTO) layer was deposited via LPCVD to serve as a hard mask for subsequent wet etching. The wafer is then patterned lithographically on both sides with photoresist SPR 220.7 ($10\text{ }\mu\text{m}$ thickness) and transferred to the Si_3N_4 hard mask by RIE. Lastly, deep reactive ion etching (DRIE) is conducted to etch through the silicon wafer, and isotropic KOH silicon etching is

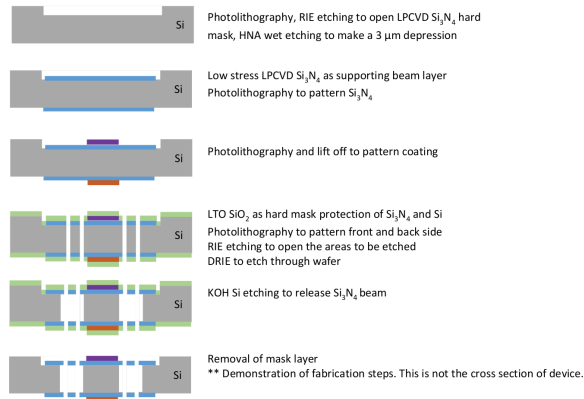


FIGURE 4. Outline of nanofabrication process of silicon mechanical resonators.

employed to release the silicon nitride beams with the oxide hard mask, which is then removed as a final step. Figure 5 shows images of the released beams and mass of our mechanical oscillator.

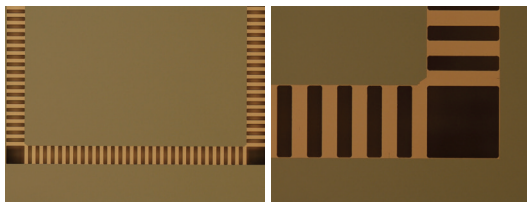


FIGURE 5. Images of the released beams and mass of our mechanical oscillator.

OPTICAL DETECTION SCHEME

In order to build an integrated high sensitivity optical sensor, a plane-concave Fabry-Pérot cavity is assembled between the flat surface of the moving test mass and a hemispherical input mirror with a radius of curvature around 300 μm . The depth of the concave mirror is about 220 μm . Typical cavity lengths between 230–270 μm are currently being tested in order to maintain a stable optical cavity, yielding free spectral ranges around 600 GHz. Moderate finesse ($\mathcal{F}$) levels in the order of 1000 combined with such a large FSR make it challenging to reliably implement conventional AC detection and cavity-lock schemes, such as Pound-Drever-Hall, due to the wide cavity linewidths involved and the necessary high modulation frequencies. Feasible alternatives can be implemented by applying deep dither modulations to the cavity length or laser frequency. It is our goal to develop cavities of finesse levels higher than 10^4 in order to accomplish narrow linewidths that enable both, robust detection and locks as well as

significantly higher sensitivity.

An additional challenge resulting from such short cavity lengths is posed on both, the laser system and the cavity length actuation. Due to the large FSR, a dynamic range of several 10s to 100s of GHz may be required to tune the laser frequency to the cavity resonance. Presently, we utilized a largely tunable telecom laser with a wavelength nominally around $\lambda = 1550 \text{ nm}$ and tunable range of 50 nm. We operate at moderate optical power levels of approximately 1 mW.

The optical output of a Fabry-Pérot cavity in reflection, as a function of the laser wavelength λ can be modeled by the Airy function, describing the signal $V(L, \lambda)$ measured at the photoreceiver[5]

$$V(L, \lambda) = v_o + \gamma \frac{(1 + R)^2}{2} \left[\frac{1 - \cos\left(\frac{4\pi}{\lambda} L\right)}{1 + R^2 - 2R \cos\left(\frac{4\pi}{\lambda} L\right)} \right] \quad (2)$$

where L is the cavity length, λ is the laser wavelength, R is the net reflectivity of the cavity mirrors, γ is the signal visibility, and v_o is an offset. Given the wide tunability range of our laser, it is possible to measure a few FSR of the cavity and thus directly measure the optical cavity length L as,

$$L = \frac{c}{2 \text{FSR}}. \quad (3)$$

High sensitivity displacement measurements at levels of $10^{-14} \text{ m}/\sqrt{\text{Hz}}$ and $10^{-16} \text{ m}/\sqrt{\text{Hz}}$ have been demonstrated in similar micro-optomechanical systems with low[5] and high[6] finesse fiber-optic cavities, respectively. Figure 6 shows the different responses for low ($\mathcal{F} = 2$) and high ($\mathcal{F} = 1000$) finesse silicon cavities with a length of 250 μm .

We have been able to demonstrate stable cavities of modest finesse with our nanofabricated silicon mirrors. Figure 7 shows the response of a 170 μm long cavity with a modest finesse of 14, which yields a FSR of 890 GHz and a cavity linewidth of 64 GHz.

Furthermore, in spite of such large cavity linewidths, we have demonstrated closed-loop locked operation of these cavities by applying a low frequency deep modulation to the cavity length at 1 kHz via a piezo-electric transducer. A data chart illustrating this operation mode is presented in Figure 8.

A fully assembled MEMS optomechanical accelerometer is in progress. It is expected that

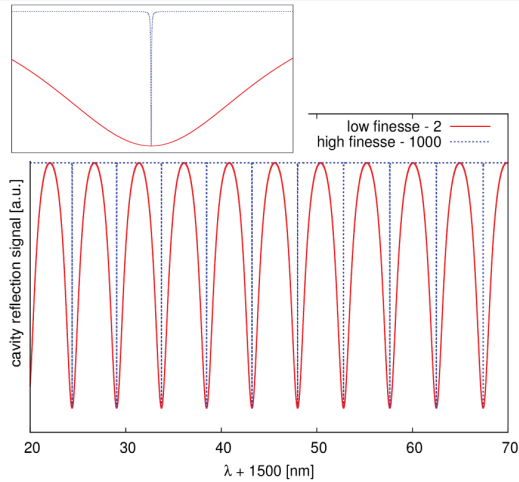


FIGURE 6. Response of silicon cavities with a length of $250\mu\text{m}$ and finesse levels of 2 in red, and 1000 in blue.

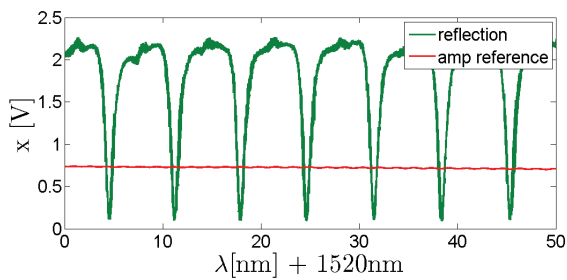


FIGURE 7. Response of a $170\mu\text{m}$ long cavity using our hemispherical Si mirrors with a gold coating.

a first acceleration measurement with these devices can be demonstrated within the next few months. Moreover, our current data and cavity responses indicate that it will be possible to achieve the targeted displacement resolutions at levels of $10^{-15}\text{ m}/\sqrt{\text{Hz}}$, by comparison to the results published on similar devices[5, 6].

Hence, we fully anticipate to meet our target accuracy below 10^{-3} for acceleration measurements with portable chip-based optomechanical standards.

DISCLAIMER

Certain commercial equipment, instruments, processes or materials are identified in this paper for completeness. Such identification does not imply a recommendation or endorsement by the National Institute of Standards and Technology.

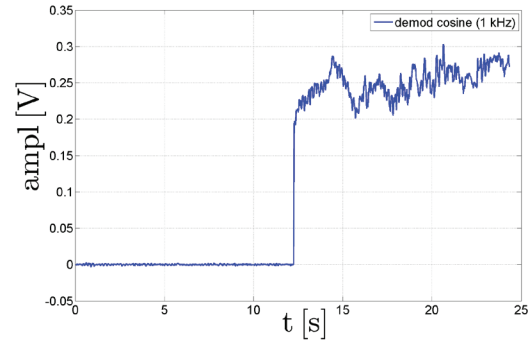


FIGURE 8. Data chart illustrating a 1 kHz low-frequency closed-loop cavity lock with a cavity linewidth of 64 GHz.

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SUB-WAVELENGTH DIFFRACTION GRATINGS: A NOVEL PLATFORM FOR OPTOMECHANICS

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ABSTRACT

Diffraction gratings with periods smaller than the wavelength of the incident light have recently emerged as a novel platform for a host of optical devices. We design and fabricate such “high contrast gratings” in silicon nitride, creating not only the world’s lightest high-reflectivity mirror, but simultaneously a very high quality mechanical resonator. We have integrated the structures into Fabry-Perot cavities, enabling sensitive readout of the mechanical motion, along with the ability to control the mechanical dynamics via the radiation pressure of the intracavity field. We are able to optically cool hundreds of mechanical modes simultaneously, reaching effective temperatures in the vicinity of 1 K. In a complementary regime, we overcome the intrinsic mechanical damping of the grating mirror by means of optically-furnished gain, and study multimode dynamics in a “phonon laser.”

AN UPGRADED DATA ACQUISITION AND DRIVE SYSTEM AT THE NANOMETER COMPARATOR

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ABSTRACT

This contribution describes an upgrade of the data acquisition and motion control of the PTB vacuum length interference comparator. The drive system based on a linear motor was extended by using a secondary motion controller, which drives a Lorentz actuator and is traced back on vacuum displacement interferometer data. First measurement results with a bandwidth of 100 Hz show that the new drive system leads to a positioning noise (RMS) of about 0.2 nm. The upgrade of the machine resulted in improved speed stability and increased position stability and enabled us to reduce the repeatability of measurements with an incremental encoder system to less than 0.1 nm.

MOTIVATION

The most demanding requirements on the measurement uncertainty for position and size of nanoscale features are defined today by lithography technologies, as described in the regularly updated International Technology Roadmap for Semiconductors (ITRS) [1].

In order to calibrate the distance of lithographic features and high performance displacement sensors, the PTB has developed the so-called Nanometer Comparator (NMC). This machine is the reference measuring machine of the PTB for the calibration of line scales [2], incremental encoder systems [3] and photo masks [4]. Previous research of our group had demonstrated that these devices under test can be characterized with repeatabilities down to the sub-nanometer range and uncertainties in the single-digit nanometer range.

During the measurement process the device under test is placed on a measurement slide, which is guided by air bearings and is moved by a drive system relatively to the particular probing system. This movement is tracked by a vacuum length interferometer and two angle interferometers. The linear drive and the slide have their own guidance and were formerly connected by a driving rod, which contained two thin places to prevent mutual interferences. The

old arrangement is shown schematically in Fig. 1. In spite of the motion control by the linear motor, the measurement slide has been subject to vibrations in the nanometer range. These mechanical vibrations of the measurement slide still appeared, suppressed in size, in the difference signal of the device under test and interferometer although the signals were sampled simultaneously. In addition, the reduced stiffness of the connection between the linear drive and the measurement slide limited the usable control bandwidth to a few 10 Hz. Furthermore, the restricted control rate of the motion controller of 2.25 kHz appeared not to be adequate for a highly dynamic motion control system.

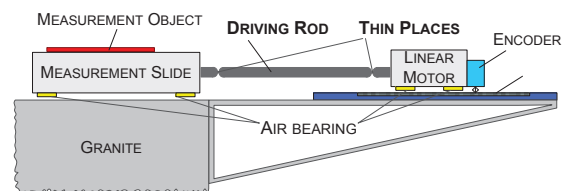


Figure 1. Old drive system on NMC with thin places.

Therefore, we decided to replace the thin places with a Lorentz actuator. In this regard, the Lorentz actuator represents a mechanical decoupling because its air gap is able to accept the guiding distortions introduced by the drive.

The position of the slide is supposed to be traced directly back to our heterodyne interferometer data and not the encoder feedback data, which can improve the positioning repeatability of the device under test. Previous work of our group demonstrated that phase variations [5] as well as interferometer nonlinearities [6] can be detected down to the level of a few picometers. In addition, preceding work in the field of precision control of air bearing stages have revealed that the performance can be improved using fast motion controllers [7,8] and using high precision position feedback signals [9-11] as well.

On the basis of these findings, we have built up a test setup using components, which were also intended for a later integration into the NMC. It

UPGRADE OF NANOMETER COMPARATOR

Developments on the NMC have resulted in a continuously improving performance [13,14]. Recently, the NMC has been upgraded to provide straightness measurements using a Zerodur sample carriage equipped with a mirror at one sidewall and an interferometer system consisting of three measurement beams [14]. This structural modification led to improvements with respect to stability and repeatability but also required a new position sensing system to track up to eight interferometer axes simultaneously. However, the projected integration of a Lorentz actuator required changes on the motion control and on the position sensing concept, as well. In this regard, we have taken advantage of our FPGA-based phase meter [5] to detect precisely the position and transfer the related data fast and deterministically to a dedicated FPGA-based motion controller. The principle layout in Fig.2 illustrates the signal flow of the implemented data acquisition and motion control systems, which will be discussed in the following sections.

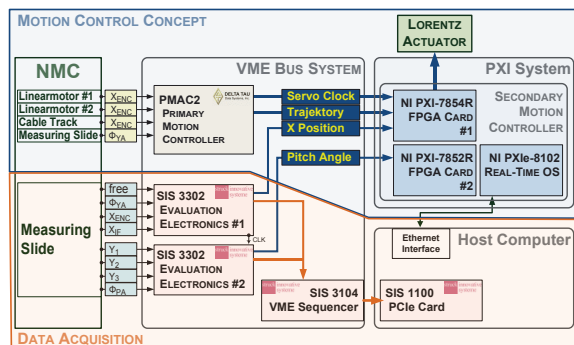


Figure 2. Principle layout of the upgraded data acquisition and control concept, which are both highlighted in orange and blue, respectively.

Advancements in motion control on NMC

The NMC motion control had to be adapted after the Lorentz actuator was integrated. In order to avoid a complete redevelopment of the motion control, we decided to make a reasonable compromise by leaving the coarse position control at the commercial motion controller (Delta Tau PMAC2) with a resolution of 5nm and added a fine position control based on a

commercial FPGA system of National Instruments.

The basic principle of the new motion control system is depicted in Fig.3.

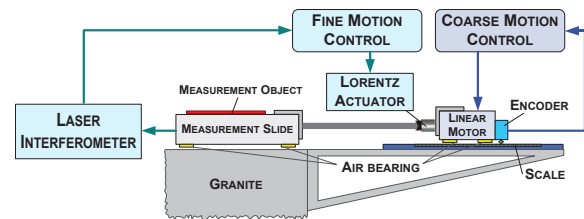


FIGURE 3. Basic principle of the implemented servo concept consisting of a fine motion control loop operated with vacuum interferometer feedback signals and a Lorentz actuator and coarse control loop driven by a linear motor and fed back by linear encoder signals.

The PXI system consists of two FPGA cards (NI7854R,7852R) and a Real-Time Controller (NIPXIe-8102), which are all located inside a PXI chassis (NI PXIe-1071).

A fast data transmission was used to feed the first FPGA card with position data supplied by our phase meter. In order to provide high resolving feedback signals, the transmitting position data were requested to offer a resolution down to 1 pm and a moving range of about ± 1 m. In addition, only three digital output lines were available at the FPGA-based phase meter. To meet these demands, an adapted Synchronous Serial Interface (SSI) transmitting absolute position information and featuring a bit width of 41 bit was implemented into the FPGA controller and phase meter [12].

In order to close a motion control loop a measured and reference position is required representing the position feedback signal and the trajectory signal. A parallel data bus is applied to transmit the PMAC2 trajectory data to the FPGA card, which latches the desired position derived from the servo clock signal. The trajectory data have a width of 32bit, offer a resolution of about 1 pm and a range of 5.4 mm. The data are unwrapped on the FPGA to extend the moving range and are then employed to process subdivision data with a rate of 33 MHz by a linear regression method.

Following, measured and reference position are delivered to the FPGA controller and can then be used to calculate the tracking error, which is served to a PD control method with a rate of 160 kHz. The control unit processes an actual value and outputs a voltage using its digital-to-analog converters featuring a resolution of 16 bit.

The voltage is then converted into a current using a feedback transconductance amplifier, which has been already proven in nanopositioning applications [15]. The Lorentz actuator is driving the slide based on the feedback signals derived from our vacuum interferometer.

Advancements in data acquisition on NMC

The use of our phase meter allows us to track heterodyne measuring systems with arbitrary beat frequencies as well as homodyne systems. Moreover, filter bandwidth of the lock-in algorithm of the phase meter can be easily adapted. In this regard, the beat frequency has been reduced from 20 MHz to about 2.5 MHz in order to improve the resolving capability of the phase meter.

The phase meter is capable to track motions with speeds up to $26 \text{ mm} \cdot \text{s}^{-1}$ and buffers position data with an acquisition rate of 48.8 kHz in the on-board memory for measurement purposes. The phase meter is also able to capture the actual position derived from an external trigger with rates up to 800 kHz, which is required to synchronize the acquisition to an external measuring system.

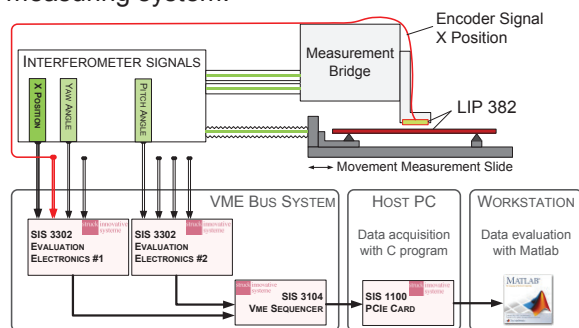


FIGURE 4. Basic principle of the measurement procedure by tracking simultaneously the displacement measuring systems with self-developed evaluation electronics.

The acquired data are then transferred over VME bus to a host computer, which is illustrated in Fig.4. This PC collects and saves the measurement data and provides the graphical user interface to operate and perform the measurements with the NMC. The measurement procedure has been changed in such a way that the data can now be acquired synchronously on one single card and not in a triggered fashion, as it was operated before by using commercial evaluation electronics [3].

Moreover, a signal propagation delay between different measuring systems connected to the

proposed electronics can easily be compensated with a timing resolution of 10 ns. In the last step, the stored data are transferred to a work station to execute the data evaluation using Matlab.

PERFORMANCE EVALUATION

The performance of the upgraded Nanometer Comparator has been recently verified by static and dynamic measurements, in which our phase measurement system acquired the position of measurement slide and device under test, a Heidenhain encoder system (Model: LIP 382). The applied commercial encoder system consists of a sensing head and a 287 mm long grating structure on a 15 mm thick Zerodur carrier. Such incremental encoder systems have turned out to be valuable and suitable tools to characterize the performance of length measuring machines [3, 16].

All results except nonlinearities measurements have been processed in the exact same manner to offer a bandwidth of 100 Hz, which corresponds to a lateral resolution of $20 \mu\text{m}$ at a moving speed of $1 \text{ mm} \cdot \text{s}^{-1}$. Furthermore, a delay of 140 ns was evaluated by reducing the difference between both measuring systems at changing motion directions and has therefore been compensated on the phase meter.

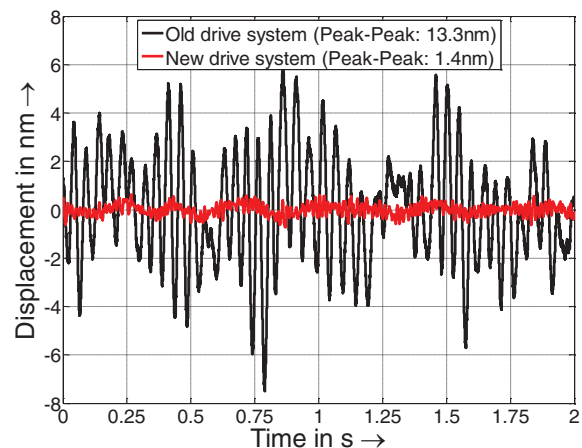


FIGURE 5. In-position measurements to compare the old drive system (black curve) and the new drive system (red curve) measured with the interferometer.

Static Measurements

An in-position measurement of the slide after integrating the Lorentz actuator, depicted in Fig. 5, shows a positioning noise (RMS) of about 0.20 nm (red curve). This is an improvement by a factor of 9 compared to the former drive

system, which was limited by its phase commutation capability. Moreover, the difference between the interferometer and the encoder system could be reduced from a value of 0.09 to 0.05 nm (RMS). This improvement is caused by the capability of the upgraded drive system to suppress superiorly mechanical vibrations stimulated previously by the linear motors.

In another investigation the quasi-stationary characteristics of the new motion control concept was verified by moving the slide, which has a weight of 150 kg and is connected to the vacuum chamber, with steps of 0.33 nm. After upgrading the drive system it is possible to move the slide with sub-nanometer precision, which is highlighted in Fig. 6.

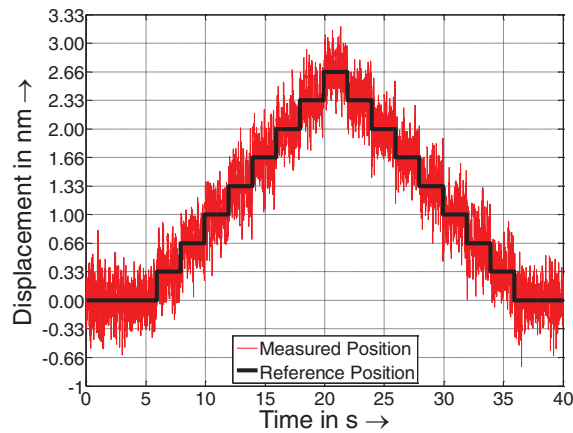


FIGURE 6. Measurement result of the new drive system performing steps with 0.33 nm width.

Dynamic Measurements

Further measurements were conducted with the encoder system at a moving speed of $1 \text{ mm} \cdot \text{s}^{-1}$ and reveal an improvement of the speed stability by a factor of 10, as shown in Fig. 7.

Six sets of measurements were consequently performed with changing orientations, in doing so each set consists of 12 measurements. A repeatability was computed for each of these sets based on variations from the common mean value of each set.

During these measurements a repeatability of less than 0.1 nm could be obtained throughout. The comparative result of one set using the old as well as the new drive concept is depicted in Fig. 8. There are variations visible differing from the mean as a function of the motion direction and exhibiting amplitudes of about 50 pm. We assume that this behavior is caused by thermal effects and can be approximated with a cubic fit. Hence, it is possible to reduce the repeatability to about 0.05 nm if such a fit is applied.

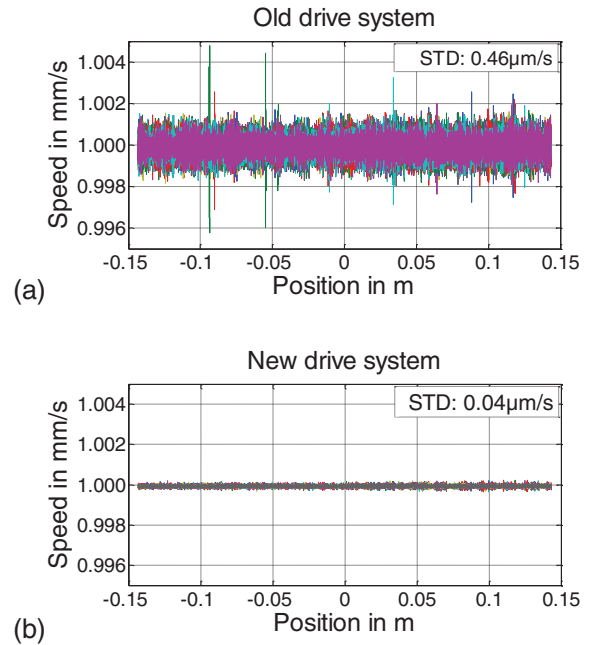


FIGURE 7. Measurement results to compare the speed stability of (a) the old drive system with thin places and (b) the new drive system with Lorentz actuator. A 10-fold reduction of the standard deviation (STD) was achieved.

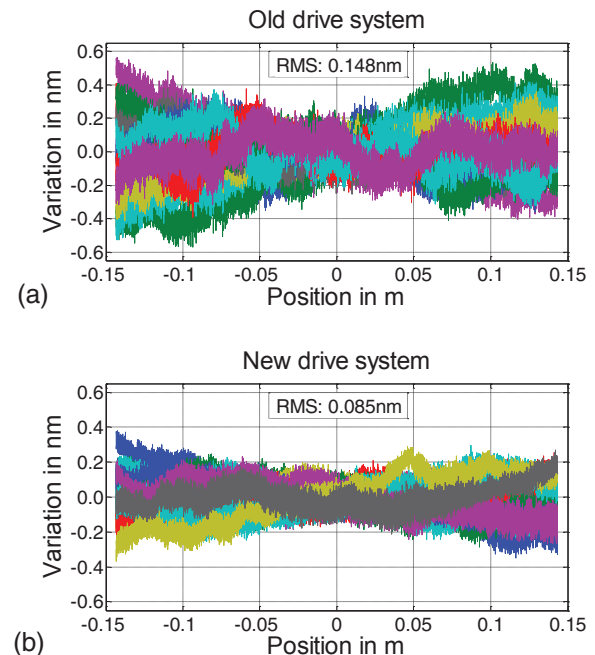


FIGURE 8. Measurement results with an encoder system to determine the repeatability using (a) the old drive system with thin places and (b) the new drive system with Lorentz actuator.

Nonlinearities

The nonlinearities of incremental encoder systems have been subject of investigations in prior publication by using scanning tunneling microscopy [17], X-ray interferometry [18] and optical interferometry under vacuum conditions [3]. It was proven that nonlinearities in the sub-nanometer range could be achieved while the evaluation electronics were applying an on-line correction.

The displacement interferometer optics have been designed to avoid nonlinearities using spatially-separated input beams and are operated by means of a single, iodine-stabilized, frequency-doubled Nd:YAG laser source.

In order to investigate directly the nonlinearities the technique of velocity scanning [19] was applied. The utilization of an incremental encoder featuring a differing signal period to the vacuum interferometer enables us to separate the nonlinear deviation of both systems. In comparison to the aforementioned analyses no averaging was applied and the slide was moved with a speed of $1 \text{ mm} \cdot \text{s}^{-1}$.

In a first step, the nonlinearities of the encoder data were compensated by applying an overall Heydemann correction [20] using a single coefficient set. In a second step we applied a stepwise Heydemann correction by using intervals of $10 \mu\text{m}$ to adapt the ellipse fit area by area. The heterodyne interferometer data were not supposed to be corrected for any nonlinear influence. The difference between both measuring systems was computed to determine a noise floor of the measurements to a value of about $0.1 \text{ pm}/\sqrt{\text{Hz}}$, which is illustrated in Fig. 9.

First, it was observed that the heterodyne interferometer exhibits an amplitude of 4.89 pm at a Doppler frequency of 3.757 kHz , which represents the first order cyclic error. Turning now to the encoder, the first and second order of nonlinearities are higher than 100 pm while executing the overall Heydemann correction [20]. In the next step, we applied the above mentioned stepwise correction method and were able to reduce the nonlinearities off-line to a value of 2.27 pm and 0.18 pm for the first and second order cyclic errors, respectively.

The analysis shows that a nonlinear influence on encoder systems can be reduced to a level of less than 5 pm by applying an appropriate off-line correction method. In addition, the nonlinearities of our vacuum interferometer were observed to be in the range of less than 10 pm without any correction.

The upgraded drive system is capable to improve the frequency domain method used to determine the nonlinearity amplitudes due to its improved speed stability. The 10-fold improved dynamic stability results in a reduction of the spectral width of the nonlinearities from 3 to 0.3 Hz compared to the old drive system. Furthermore, the improved data acquisition system allows us to determine the current nonlinearity in each data set taken.

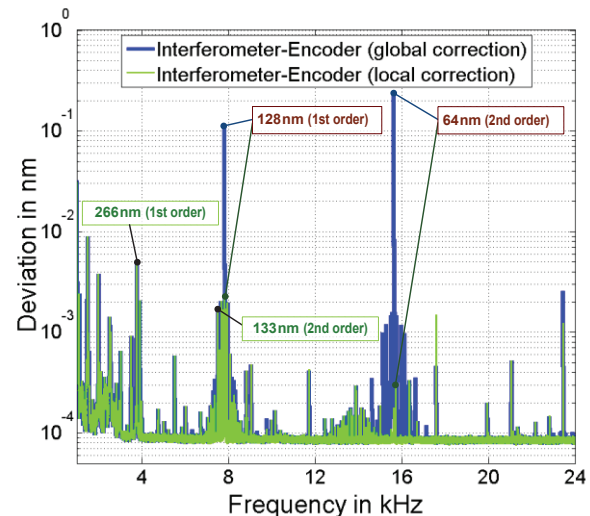


Figure 9. Spectral analysis of a comparative measurement with an encoder system pointing out nonlinearities of heterodyne interferometer without correction and device under test after applying a Heydemann correction with global method (blue curve) or stepwise method (green curve). Interferometer nonlinearities appear at periods of 266 and 133 nm , while the encoder nonlinearities coincide with a multiple of 64 nm .

CONCLUSION

This publication reports on recent developments at the Nanometer Comparator by extending the motion control with a Lorentz actuator. A self-developed phase meter provides highly precise position data to a secondary positioning loop, which was formed on the basis of an FPGA-based motion controller. The new drive system is capable to control the slide position with a positioning noise (RMS) of 0.2 nm , which is an improvement by the factor of 9 . These findings are also promising in the light of proceeding improvements at prospective line scale measurements with a CCD microscope.

Furthermore, the performance of the proposed drive system was investigated using a new data acquisition system. Repeatabilities of less than

0.1 nm were achieved during measurements with an incremental encoder system. The upgraded data acquisition system enables us to verify nonlinearities with a resolving capability of about 0.1 pm/√Hz. The encoder nonlinearities were reduced off-line to a level of less than 5 pm by applying a stepwise Heydemann correction.

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INTEGRATION OF FABRY-PEROT INTERFEROMETRY WITH A NEW CALCULABLE CAPACITOR

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INTRODUCTION

The research and development of calculable capacitors has been an intriguing topic since the discovery of the Thompson-Lampard theorem [1], which serves as a principle for realization of the SI (International System of Units) farad. The NIST calculable capacitor [2, 3], which was built in the 1970s, relates electrical capacitance to mechanical displacement via interferometry using fixed-wavelength lasers. In this article we describe our current effort on developing a new calculable capacitor, [4, 5] focusing on the aspect of Fabry-Perot (FP) interferometry using tunable lasers for precision displacement measurements.

OVERVIEW OF NEW CALCULABLE CAPACITOR

The main assembly of the new calculable capacitor is shown in Figure 1. The structure consists of four identical fixed cylinders, which serve as main capacitance electrodes, and two blocking electrodes along the central axis. The lower blocking electrode is fixed and the upper blocking electrode is movable. The motion of the upper electrode is controlled by a Physik Instrumente (PI)¹ linear actuator, which has a travel range of 51.3 mm, allowing the calculable capacitor to operate in between 0.1 pF and 0.2 pF with a net change of 0.1 pF. The cross capacitance between two opposite electrodes is then determined by the absolute mechanical displacement of the upper blocking electrode on the vertical axis.

Heterodyne interferometry is used to measure the absolute displacement between the two blocking electrodes. The upper one incorporates a fixed flat mirror and the lower one incorporates a piezo-actuated concave mirror. These two mirrors thus form a plano-concave FP cavity with a finesse of

¹Product Disclaimer: Certain commercial equipment, instruments, or materials are identified in this report in order to specify the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the equipment identified are the best available for the purpose.

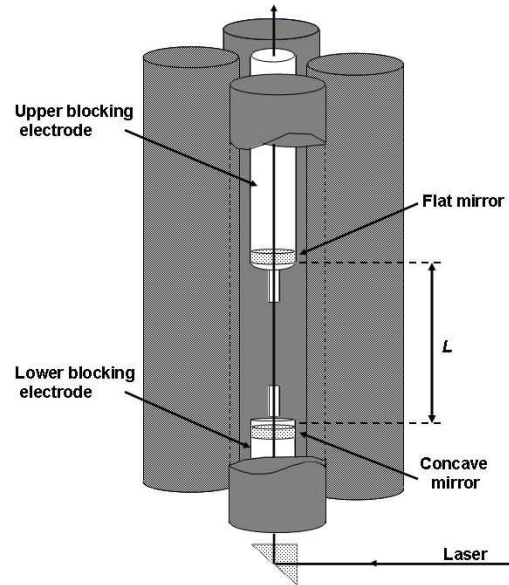


FIGURE 1. The main assembly of the new calculable capacitor. The upper blocking electrode is movable to change the cross capacitance, which is linked to the displacement measurement of the cavity length.

1000 and a linewidth of about 2 MHz. The interferometer employs two fiber-coupled 1560 nm tunable Orbits Lightwave lasers, which are frequency locked to different resonance modes of the cavity length via the Pound-Drever-Hall (PDH) technique [6] independently. Depending on the cavity length and the absolute laser frequency, the heterodyne beat between the two lasers can vary from 10 to 20 free spectral ranges (FSRs) of the cavity, corresponding to an RF frequency approximately from 11 to 19 GHz.

The metrology consists of two steps. First, we measure the FSR of the cavity by measuring the beat frequency referencing to the internal clock of a radio frequency counter ("RF" method) [7]. If we assume $\nu_1 = N(c/2L)$ and $\nu_2 = (N + M)(c/2L)$, where ν_1 and ν_2 are the absolute frequency of the two lasers, L is the cavity length, and M is the

number of spacing between the two resonance modes, then the beat frequency is given by

$$\Delta\nu = \nu_2 - \nu_1 = M \frac{c}{2L} = M \cdot \text{FSR}. \quad (1)$$

Since M is an integer, it only requires prior knowledge of a nominal length L (sub-mm uncertainty) to infer M and the FSR. The cavity length L can also be calculated using the equation above. The precision of the RF method totally depends on the stability of the beat frequency.

The second step is to infer the mode number N using the FSR obtained from the RF method, which requires the knowledge of the absolute wavelength, such that the cavity length can be calculated by the resonance condition $L = N\lambda/2$ ("optical" method) [7]. The mode number N is an integer on the order of 10^5 ; therefore, the uncertainty of the RF method needs to be less than 1 ppm to avoid any ambiguity to the last digit of the N value. The precision of the absolute wavelength measurement can be achieved down to 0.02 ppm with Bristol wavelength meter 621A, and can be further enhanced if we introduce an absolute frequency reference, i.e., a mode-locked frequency comb. With a deterministic N value, the precision of measurement is substantially increased compared to the RF method.

In reality, the calculated N value is usually not an integer. The non-integer part of N is due to several effects that change the effective cavity length, including Gouy phase shift and the penetration of the electric field into the dielectric mirror. For a plano-concave cavity, the contribution to N from Gouy phase is given by [8]

$$\delta N_{\text{Gouy}} = \frac{1}{\pi} \sin^{-1} \sqrt{\frac{L}{R}}, \quad (2)$$

where R is the curvature radius of the concave mirror. When R is large (50 cm) compared to the cavity length L , the variation of δN_{Gouy} is small as a function of the cavity length. Therefore, the non-integer part of N is fairly deterministic and consistent for all different cavity lengths.

ACTIVE CANCELLATION OF RESIDUAL AMPLITUDE MODULATION

The RF method requires the fluctuation of the beat frequency to be less than 1 ppm. Therefore, various sources in the noise budget need to be considered and sufficiently mitigated. Residual amplitude modulation (RAM) [9, 10, 11, 12],

which is due to the birefringence-related effects inside the phase modulator, is one of the common noises that appear in PDH locking. Typically, RAM is generated from the axis mismatch between the polarization of the incident light and the orientation of the modulator crystal [10], or the etalon effect that causes the light to reflect multiple times inside the crystal [9]. In either case, temperature fluctuation plays a significant role in changing the birefringence and geometry of the crystal. Therefore, RAM is usually temperature-dependent and difficult to mitigate simply by passive polarization adjustment [13]. In a fiber-coupled PDH loop, the issue of RAM becomes more significant due to the distortion of laser polarization inside the fiber. Unlike the propagation in free space, it is difficult to keep light statically linearly polarized even with polarization-maintained (PM) fiber. Random birefringence arises from mechanical stress, temperature fluctuations or the inhomogeneity of the fiber. This time-dependent variation in polarization causes a rotation to the optical axes of the electro-optics modulator (EOM) and polarizing beam-splitter (PBS), which generates polarization-rotation-induced RAM in the error signal and consequently a noisy baseline drift over time.

Several different approaches [14, 15, 16, 17] have been demonstrated to suppress the amplitude modulation in the phase modulator. In general, one can build an additional feedback loop to apply a DC bias voltage [10, 13, 18] to the EOM to compensate the phase shift caused by the natural birefringence in the crystal. This method is mostly restricted to the suppression of phase-modulator-related RAM. The servo may encounter the problem of limited dynamic range when the bias voltage is applied. The phase shift due to natural birefringence can vary by several wavelengths, which may drive the applied DC bias out of its tuning range and thus the servo requires to be manually reset on a regular basis.

In this article we introduce a simple approach to solve the issue of polarization-induced RAM without using an additional control loop. Like the active control approaches mentioned above, we also measure and demodulate the RAM before the light enters the cavity. Instead of sending this signal to the DC port of the phase modulator, we combine this signal with the standard PDH error signal to cancel out the RAM. The experimental

setup to verify the RAM cancellation scheme is illustrated in Figure 2. For simplicity only the frequency stabilization loop for one laser is shown here.

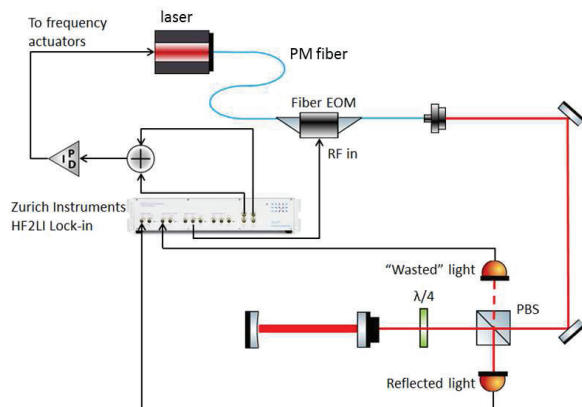


FIGURE 2. The experimental setup to demonstrate the RAM cancellation scheme. The picture of HF2LI lock-in amplifier is courtesy of Zurich Instruments.

When propagating through the PM fiber, the light field is generally elliptical polarized due to random birefringence. Therefore, after the fiber EOM the light includes both the regular phase-modulated component and the unmodulated component that is perpendicular to the optical axis of the crystal. The key part of this setup is that we simultaneously measure the “wasted” light that is reflected at the PBS with another photodetector. Since there is a time-dependent angular mismatch between the polarization of the phase-modulated light and the optical axis of the PBS, this “wasted” light includes both the sine component of the phase-modulated light and the cosine component of the unmodulated light. If we demodulate this signal with the same RF modulation signal and add it to the regular PDH error signal, the combined baseline of the error signal is time-independent and completely insensitive to the distortion of the polarization due to stress or temperature fluctuations. Also, the error signal on resonance will not be affected in this combination since the “wasted” light does not go into the cavity. In reality, the two signals from each photodetector may have different gain factors and may not be exactly 180° out of phase, which requires further adjustments at the lock-in amplifier and limits the performance of cancellation. Figure 3 shows the long-term measurement results of the reflected light, wasted light and the error signal combined from them. Here the photodetec-

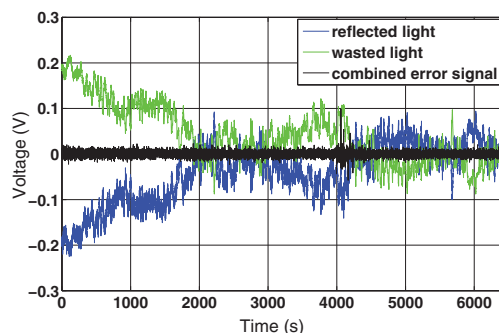


FIGURE 3. Time series of the photodetector signals and their combination. Most of the time-dependent drift due to RAM is canceled in the combination.

tor signals of the reflected and wasted lights are already demodulated by the same RF signal from the lock-in amplifier. The outputs of the lock-in amplifiers, as shown in the figure, are quite identical in magnitude and 180° out of phase. This figure clearly shows how the error signal baseline drifts over time in a standard PDH setup and the effectiveness of our approach, which has reduced the standard deviation of the error signal from 63 mV to approximately 4 mV in this measurement.

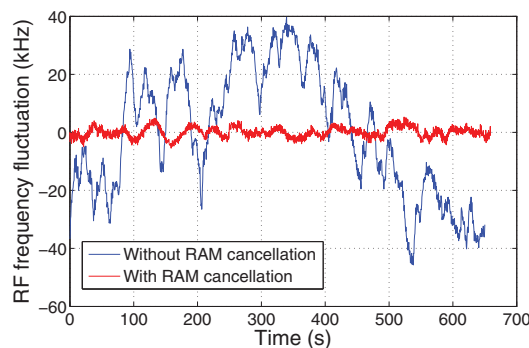


FIGURE 4. Frequency fluctuations of the RF beat between two cavity stabilized lasers.

The effect of the RAM cancellation on frequency stabilization is demonstrated in Figure 4. The red and blue curves represent the fluctuations of the beat frequency when the RAM cancellation scheme is and is not present in the setup, respectively. Both cases have the same nominal RF frequency at 14.65 GHz. The fluctuation of the blue curve actually corresponds to half a cycle of a 0.8 mHz oscillation, which is correlated to the periodic temperature fluctuations of the environment. The peak-to-peak value of the blue curve is more than 80 kHz, which corresponds to an un-

certainty of 6 ppm. With this kind of uncertainty in the RF method, the calculated N value for the optical method spreads out to larger than 1 such that it is difficult to infer a deterministic value. In comparison, the RAM cancellation scheme has effectively eliminated the 0.8 mHz noise due to temperature fluctuations as well as RAM-related noise at other frequencies. The standard deviation of the RF noise is reduced to 2 kHz and corresponds to splitting the 2 MHz linewidth Fabry-Perot fringes by a factor of 1000. We measured the wavelength of one of the lasers simultaneously using the Bristol 621A wavelength meter. The N values were calculated for each RF frequency and its synchronized wavelength data. The distribution plot of N is shown by Figure 5, which yields a deterministic N (222966.2) and a Gaussian distribution.

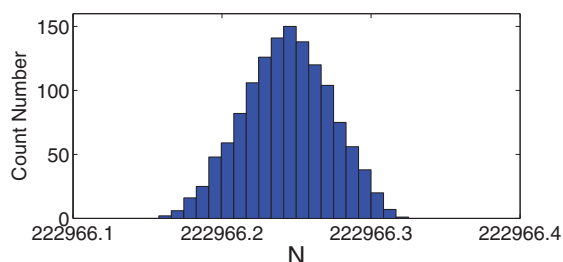


FIGURE 5. Distribution of N values obtained from RF frequency and wavelength measurements.

As a result, the relative precision of the FP interferometry using the optical method is a few parts in 10^8 , currently limited by the precision of the wavelength meter.

CHARACTERIZATION OF THE NEW CALCULABLE CAPACITOR

Using the FP interferometry described above, we have started to characterize the new calculable capacitor. An Andeen-Hagerling (AH) 2700A capacitance bridge is used for capacitance measurements. Although the capacitance resolution is about 0.8 aF at 1 kHz according to the manufacturer's specifications, we have achieved a resolution below 0.1 aF through averaging.

Alignment

The fractional difference between the two cross capacitances is required to be on the order of 10^{-4} or less, and the position of main electrodes is required to have a deviation no more than a few 100 nm throughout the travel range [19]. This alignment is ensured by measuring the capacitance C_1 , C_2 , C_3 and C_4 between a ring sensor on

the upper blocking electrode and each main electrode individually. The reference range for alignment is from 5 mm to 35 mm, partly because the sensor capacitance has large deviations near the upper end due to leakage. Figure 6 shows the sensor capacitance throughout the linear actuator range. The largest deviation occurs in C_1 and C_3 between 5 mm and 10 mm, which is about 1 fF, corresponding to about 500 nm misalignment. The average of two opposite sensor capacitances (C_1 and C_3 , C_2 and C_4) has an even smaller deviation of a few parts in 10^5 , indicating that the motion of the upper blocking electrode is the dominant error source in the sensor measurement.

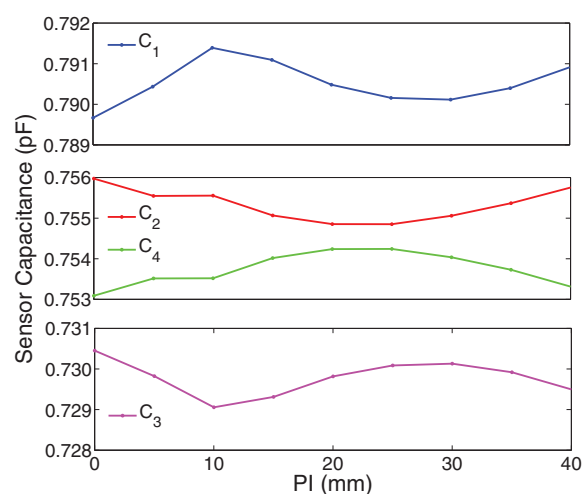


FIGURE 6. The variation of sensor capacitance between upper blocking electrode and individual main electrode from 0 to 40 mm.

Stability

The stability of the new calculable capacitor has been investigated at two end positions of 0.1 pF and 0.2 pF. Figure 7 demonstrates the drift of the cavity length from the position at 0.2 pF over the course of one hour. The curve shows the capacitance change calculated from the measured displacement of the upper electrode. Since the measurement started right after the upper blocking electrode was moved to the top position, the curve shows a typical relaxation process. The measurement indicates that within one hour, the cavity length drifted about 160 nm, causing the laser wavelength to drift 14 pm. The capacitance was recorded and (not shown) follows the cavity relaxation within the resolution of the AH bridge.

Repeatability

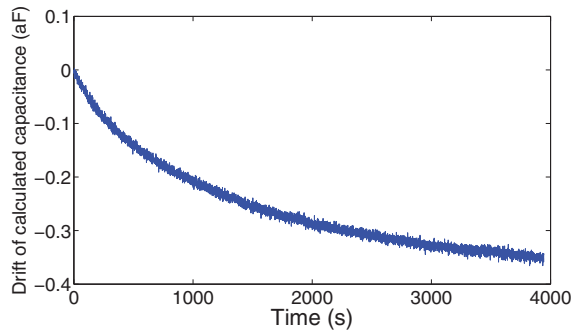


FIGURE 7. Drift of the cross capacitance, calculated from the optical length of the free-running cavity.

The repeatability of the system reflects that the interferometer readout closely follows the measured capacitance. Each time the PI linear actuator is moved to the same encoded position, the real displacement of the upper electrode can vary by up to 1 micron due to mechanical factors like non-linearity and hysteresis. For example, when we measure the cavity length and capacitance at the two end positions of 0.1 pF and 0.2 pF with multiple travels back and forth, the peak-to-peak fluctuation of the cavity length is close to 1 micron, corresponding to a capacitance variation of more than 1 aF. Since the cavity length is dynamically measured, the fluctuations due to the linear actuator can be removed. Figure 8 shows the residual fluctuations of the system at the top and bottom position after repeated travels, where ΔC is given by the difference between the calculated capacitance based on the interferometer readout and the measured using the AH bridge. This test indicates that the repeatability of the calculable capacitor is below 0.1 aF.

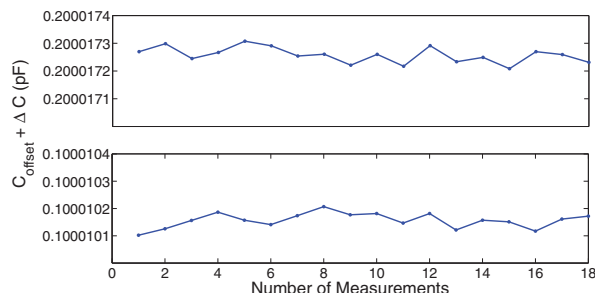


FIGURE 8. Repeatability measurement at the 0.1 pF (bottom) and 0.2 pF (top) position.

CONCLUSION AND FUTURE WORK

In summary, we have made significant progress in development of the new calculable capacitor. In particular, we have demonstrated a simple approach that is capable of suppressing the polarization-induced RAM in the PDH setup. This RAM cancellation method can avoid certain problems in the servo control such as the limited tunable range of the actuator and can be easily implemented in any PDH setup. The precision of the RF method now reaches the ppm level, enabling us to measure the absolute displacement to a few parts in 10^8 . We have also investigated the alignment, stability and repeatability of the new calculable capacitor. We are now in the process of checking other error sources, including the non-linearity of the capacitance bridge and the cosine error in optical alignment, to complete the characterization of the system.

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The NIST watt balance, the new SI, and canonical ideas of precision

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INTRODUCTION

The electromechanical experiment known as the watt balance was conceived by Kibble at the National Physical Laboratory as an evolution in the realization of the ampere [1], but it is now poised to become one of the primary experiments used to realize the unit of mass from a variety of fixed, fundamental constants in a proposed new International System of Units, or the new SI [2]. Just as the unit of length was once the space between two scratches on a metal bar in Paris, and now can be derived from the speed of light c , the unit of mass will cease to be a cylinder of platinum iridium in a vault in Paris, and will instead be derived using a fixed value of the Planck constant h .

In this brave new SI, it can be argued that we have finally succeeded in completing a long, sought-after break from artifacts and moved on to a promised land of direct access to a realization of any unit at any scale, where and however one might choose. Interestingly, there is perhaps no experiment that better illustrates the challenges, pitfalls, and ultimate rewards that might be achieved in such a new system of measurement than a watt balance.

It has been observed that a precision watt balance is in effect a National Metrology Institute (NMI) writ small. The experiment places such demands on the precision with which one must determine all the measurands that effectively each measurand is a realization of its respective unit (overall relative uncertainty must be less than 50 ppb). So in order to operate a watt balance, one must first master the realization of the meter, the second, the ohm, and the volt.

The realization of these units of measure is a first necessary step. The instrument, its mechanics, optics, electronics, data acquisition, and controls must further achieve a harmony of design, construction, operation, and execution such that the diverse optical, mechanical, and electrical systems combine to yield a realization of the unit of mass that can be trusted. Thus, in

addition to being its own NMI, a watt balance is also an exemplar of what we might phrase as the canonical ideas of precision. Here, I take inspiration from Jones [3], expressing his ideas canonically in terms of the following broad categories:

- Differentiation: the quest for sensitivity
- Symmetrization: nature lends a hand
- Stabilization: imposing quiet
- Repetition: the power of averaging

This paper reviews the basic principles of a watt balance experiment in the context of these canonical ideas applied to the NIST-4 watt balance illustrated in Figure 1.

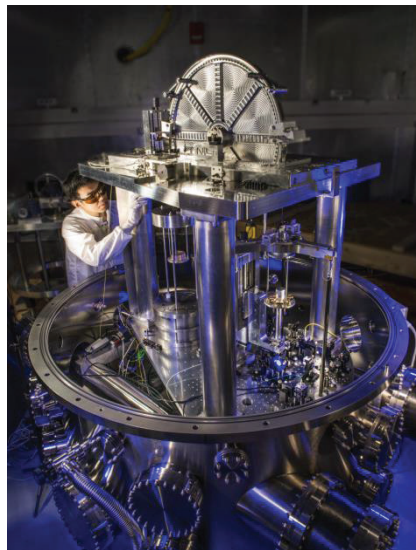


Figure 1 The NIST-4 watt balance.

WATT BALANCE PRINCIPLES

A watt balance is very similar to an ordinary balance. The NIST-4 example shown schematically in Figure 2 uses a wheel pivoting on a knife edge as its main weighing and motion mechanism, a weighing pan suspended from the wheel, and a moving coil actuator to apply compensation forces to hold a null position, just as in a normal compensation balance.

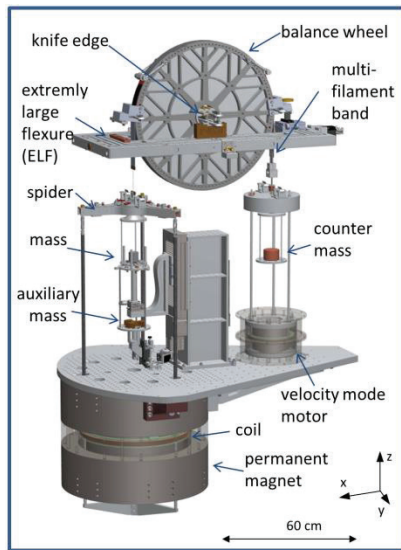


Figure 2 Schematic drawing of the NIST-4 watt balance.

When NIST-4 is used as a weighing instrument, dc current I is fed back through a coil in a magnetic field and produces the force that nulls the position of the balance mechanics and exactly balances the gravitational force acting on the test mass m . If everything is constructed properly the balance of forces can be expressed as $BLI=mg$, where B is the magnetic flux density, L is the wire length in the coil, and g is the local gravitational acceleration.

Clearly, if one has measured the local gravity, g , the current I , and the geometric factor BL , it is straight forward to compute the quantity of mass resting on the balance. In fact, it is possible to realize a unit of mass in terms of fundamental constants using this scheme, and such a weighing is often referred to as an electronic kilogram [4]. The technique is a *mise en pratique* for mass in the planned revision of the SI [5].

A CANON OF PRECISION

At its heart, the watt balance is *simply* a problem in precision instrument design and construction. The point of the experiment is not to test our knowledge of the physical universe, but rather to exploit this understanding, executing a carefully orchestrated series of measures to quantify a macroscopic mass in terms of a compact collection of invariants.

In order to cast this problem within the framework of the “canon” I mentioned in the introduction, let’s first agree on what we mean

by an instrument. Here again, Jones offers guidance [3], and Chapter 9, Some Considerations in Instrument Design begins with a definition of an instrument according to Maxwell:

Everything which is required to make an experiment is called Apparatus. A piece of apparatus constructed for the performance of experiments is called an Instrument. Apparatus may be designed to produce and exhibit a particular phenomenon, to eliminate the effects of disturbing agents, to regulate the physical conditions of the phenomenon, or to measure the magnitude of the phenomenon itself.

From which Jones develops that the basic and somewhat obvious principle aim of instrument design is to maximize the sensitivity to a specific phenomenon, or stimulus, while minimizing the sensitivity to all other “disturbing agents”.

The management of sensitivity, or DIFFERENTIATION, is the soul of instrument design, in my opinion, and rightfully appears at the top of my Jones inspired canon. Put another way, everything is a game to differentiate signal from noise, and the remaining parts of the canon are basically in the service of this essential function of an instrument. I include the idea of SYMMETRIZATION as a catchall for the ways in which natural geometries, common mode rejection, reversals, and scaling can be exploited in our quest for signal differentiation. STABILIZATION addresses Maxwell’s concern to regulate the physical conditions of the phenomenon and instrument. REPETITION acknowledges that the phenomenon and the instrument both have a certain random character, and that averaging can allow us to probe beneath this apparent noise to reveal a deterministic core.

These canonical ideas are hardly exhaustive and admittedly abstract, but with the remainder of the paper I hope to provide a few glimpses of their utility through the example of the NIST-4 watt balance, focusing mainly on DIFFERENTIATION and SYMMETRIZATION.

DIFFERENTIATION

As obvious as it may sound, the first step in designing an instrument is to decide what “phenomenon” or “stimulus” you want to

differentiate from the rest of the universe. With this in mind, it is useful to revisit the basic operation of a watt balance.

The watt balance measurement equation so far is, $BLI=mg$, which mathematically expresses the balance of electromagnetic and mechanical forces acting on the balance mechanism. Our desire is to use this balance of forces to ascertain the unknown magnitude of a mass placed on the balance to within a few parts in 10^8 . We will do this by placing the object on and off of the balance pan. As observed previously, this means we must know the values of g , I and BL at the weighing conditions so that we can solve for a value of m . The change in current arising from the two conditions, mass on and mass off, is an obvious differentiation problem.

For the sake of brevity, accept that measuring g , and I , is fairly straight forward, and that measuring BL is not. It turns out that the way to measure BL is to infer it by taking advantage of SYMMETRY in the physics of an electromagnet. Lorentz, Faraday, and ultimately Maxwell have shown us that the same coil of wire in a fixed magnetic field will function both as an electric motor and as an electric generator. Exploiting the latter, BL can be precisely calibrated in terms of length, time, and electrical standards by moving the coil through the magnetic field while measuring its velocity v_z and the voltage U induced across the coil, i.e., $U=BLv_z$.

Having measured U and v_z in a separate moving coil experimental mode we can solve for the BL factor [1] of the coil magnet system, which can then be used to compute the mass during the weighing. Measurements from the two operational modes (weighing and moving coil) also allow one to compute a virtual comparison of electrical power to mechanical power, $UI=mgv_z$, both measured in watts, hence the name watt balance. A watt balance is therefore two instruments rolled into one, alternating between a precision weighing instrument and a sophisticated tool for measuring its own magnetic field properties.

The central differentiation problems of the instrument emerge from considering these two operating modes, weighing and moving coil. First, the balance must function as a balance. It must differentiate the applied inertial load from all other forces acting on the balance during the weighing mode with the *precision* expected of a

primary mass artifact, which is presently on the order of a few tens of micrograms for a mass of nominally one kilogram, or a few parts in 10^8 . Second, the instrument must function as a gross motion stage. It must translate the coil through the magnetic field and faithfully differentiate the BL factor at the weighing position from the BL factors at all other locations and spatial configurations of the coil and magnet. As one might expect, the absolute magnitude of BL must be determined to within a few parts in 10^8 in this moving coil mode.

Providing consistency between weighing and moving coil modes has proven to be the most difficult challenge faced by watt balance designers and is why no two watt balances yet are alike. Each designer has so far found their own answer to the question: How can I construct the instrument so that BL in the moving mode is exactly the same as BL in the weighing mode? Should the balance and translation mechanism be one and the same? Or separate devices?

Kibble and Robinson analyzed this problem recently [6], developing an analytical framework that reveals how geometry, symmetry, and constraints influence these differentiation problems. Rather than a single degree of freedom, they treat the coil of a watt balance as a rigid body with six degrees of freedom, so that it has three orthogonal components (v_x , v_y , v_z) of its linear velocity $\mathbf{V}$ corresponding to displacements along coordinates x , y , and z , and three orthogonal components (ω_x , ω_y , ω_z) of its angular velocity $\mathbf{\Omega}$ corresponding to rotations θ_x , θ_y , and θ_z about each respective coordinate, as indicated by the subscripts. The z coordinate is aligned along the gravitational normal.

During weighing mode, current I flowing in the coil and linked by a magnetic flux Φ produces electrical force $\mathbf{F}$ and torque $\mathbf{\Gamma}$ on the coil:

$$\mathbf{F} = I \left(i \frac{\partial \Phi}{\partial x} + j \frac{\partial \Phi}{\partial y} + k \frac{\partial \Phi}{\partial z} \right) = iF_x + jF_y + kF_z \quad (1)$$

and

$$\mathbf{\Gamma} = I \left(i \frac{\partial \Phi}{\partial \theta_x} + j \frac{\partial \Phi}{\partial \theta_y} + k \frac{\partial \Phi}{\partial \theta_z} \right) = i\Gamma_x + j\Gamma_y + k\Gamma_z \quad (2)$$

where i , j , and k are unit vectors along coordinates x , y , and z , and $BL = \frac{\partial \Phi}{\partial z}$. The

electrical forces and torques are balanced by mechanical forces and torques that give rise to a static equilibrium. The coil is suspended from a mechanism that has stiffness along each of these coordinates, so that the mechanical forces and torques F_x , F_y , F_z , Γ_x , Γ_y , and Γ_z arise as this structure deforms and displaces its center of mass in response to the applied electrical load. The mechanical force component F_z varies during mass exchanges, and can be expressed as $tare \pm mg + err$, where *tare* is the tare mass and includes the mass of the coil and its suspension structure, and *err* refers to unwanted forces, such as balance hysteresis, associated with the weighing mechanism.

Equations (1) and (2) show that in general the coil can exert forces and torques not only along the z axis, but in off-axis directions. Similarly, in the moving coil mode, if the current supply is disconnected, and the coil is open circuited and moved with linear velocity V and angular velocity Γ , a voltage U is induced across the open terminal pair, where

$$U = - \left(v_x \frac{\partial \Phi}{\partial x} + v_y \frac{\partial \Phi}{\partial y} + v_z \frac{\partial \Phi}{\partial z} + \omega_x \frac{\partial \Phi}{\partial \theta_x} + \omega_y \frac{\partial \Phi}{\partial \theta_y} + \omega_z \frac{\partial \Phi}{\partial \theta_z} \right) \quad (3)$$

Equation (3) shows that off-axis error motions of translation and rotation will induce spurious voltages beyond that arising due to motion along the z axis.

If the magnetic flux and coil geometry are identical between the two modes, a virtual balance of electrical and mechanical power (watt balance) can be evaluated at the weighing position to yield:

$$UI = F \cdot V + \Gamma \cdot \Omega \quad (4)$$

or

$$UI = (F_x v_x + F_y v_y + F_z v_z + \Gamma_x \omega_x + \Gamma_y \omega_y + \Gamma_z \omega_z) \quad (5)$$

The phenomenon we wish to differentiate on the right hand side of Equation (5) is the product $F_z v_z$ and the signal that is detected to represent this is UI . Everything else is unwanted disturbances. Next, SYMMETRIZATION is exploited to isolate the product of $F_z v_z$,

SYMMETRIZATION

In order for the product UI to represent $F_z v_z$ with precision adequate for realization of mass, we must contrive a means to accurately measure each off-axis term and subtract it from the product UI (this is an example of a COMPENSATION based approach, a canonical precision idea I somehow failed to mention earlier) or we must construct the balance such that the ratio of each off-axis term to $F_z v_z$ is less than a few parts in 10^8 . In practice a combination of these two approaches is used.

Minimizing the off-axis terms is simply good practice, and there are essentially three ways this may be accomplished: (1) diminish the five off-axis forces, (2) diminish the five off-axis velocities, or (3) reduce both off-axis forces and velocities to diminish the off-axis product of Fv and then measure and compensate any that remain. For example, if

$$\frac{F_x}{F_z} = 10^{-4} \quad \text{and} \quad \frac{v_x}{v_z} = 10^{-5}$$

then,

$$\frac{F_x v_x}{F_z v_z} = 10^{-9}.$$

The off-axis x and y forces, as well as all the torques, can be minimized using symmetry in the magnet and coil. Off-axis forces are proportional to the gradient of magnetic flux in Equation (1). For a circular coil in a radial magnetic field, the flux gradients on opposing sides of the coil are symmetric and balanced, so that in principle the forces in the plane cancel. If the coil is also centered in the radial field, there will be no torques. The challenge, of course, is to construct and align the coil and magnet such that cancelation is complete. A perfectly radial magnetic field with a uniform vertical magnetic flux gradient is difficult to achieve, posing great challenges for the manufacturing of the pole pieces [7]. The best available precision will likely not achieve relative uncertainties in the critical dimensions better than parts in 10^5 . Furthermore, material properties of the magnets and pole pieces are likely to vary spatially on the order of a few parts in 10^4 . In practice, residual asymmetries tend to lead to off-axis forces that are on the order of $10^{-4} \times F_z$. Further cancellation is still possible, since asymmetries in the coil and magnet will offset each other for certain orientations of the coil in the field.

The apparatus suspending the coil in the magnet can be constructed so that the coil orientation is adjustable and that coil motions are only lightly constrained in the off-axis directions. As the coil is alternately energized and de-energized, the residual forces and torques experienced by the coil cause displacements of the coil from its static equilibrium that can be detected and measured. The gross orientation of the coil in the magnetic field may then be readjusted in an attempt to minimize these offsets.

On NIST-4, the coil is suspended using a series of elements shown in detail in Figure (3). Beginning at the wheel, a thin multi-filament band rolls off tangent to the rim. This band transitions to a multi-filament cable used as a rotational decoupler that allows the lower elements to freely rotate about θ_z . This cable supports a dual gimbal flexure. The dual gimbal supports both a rigid spider and the mass pan. Both the mass pan and spider are pendulums that share the same terminus, since the two gimbals are nested and have the same rotational center within manufacturing tolerances. The coil hangs off the rigid spider, suspended from three rods. The spider, rods, and coil create a cage-like frame with small flexure cubes at the joints, so that the structure can shear as a parallelogram in the x,z and y,z planes. In the end, even after careful alignment, off-axis forces will likely persist at the level of a few parts in 10^5 , leading us to consider how best to diminish the off-axis velocities. Existing designs exhibit two fundamental approaches.

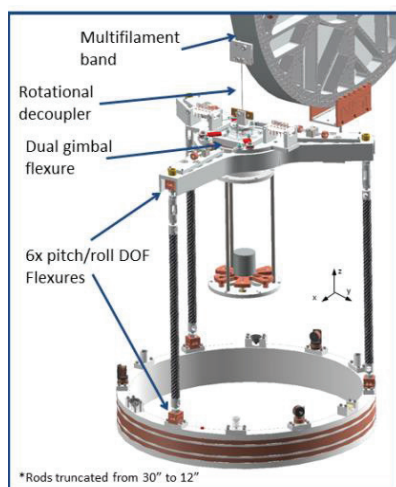


Figure 3 Coil and mass suspension system of the NIST 4 Watt balance

On the one hand, some recent designs address the functional requirements of translation and weighing using two separate mechanisms. These schemes often use commercially available mass balance mechanics for the weighing mode, and then employ a translation system linked serially between the coil and balance to achieve the several millimeters of linear travel necessary for the coil motion. A challenge in such schemes is to integrate the balance and translation systems so that the transition between weighing and moving happens without disturbing the orientation of the coil. Sometimes, the entire combination of coil and balance mechanics is translated in an effort to resolve this issue.

On the other hand, some recent designs satisfy the requirements of translation and weighing using one mechanism. The challenge in this scheme is to construct a mechanism that is essentially a frictionless translation stage. Flexure based solutions are appealing, and Kibble and Robinson [6] make a compelling case for creating an instrument where the balance mechanism and coil reduce to a single-degree-of-freedom, rectilinear spring. The idea is that if the coil and balance form a single coordinate, it will assure consistency between weighing and motion, and could in principle completely relax the need to minimize the off-axis flux gradients. Such a constrained design can certainly help minimize the off-axis velocities, but it is still a considerable challenge to create mechanisms where parasitic motions are only a few parts in 10^8 of the primary motion.

NIST-4 continues the approach that was successful on NIST-3. The balance mechanism is again a wheel that pivots on a knife edge bearing. The multi-filament band rolls off the wheel from which the coil is suspended using the spider and frame discussed in the previous section. The scheme allows the coil and mass pan to hang naturally along the gravitational axis and to self-align, preserving the symmetry of the system even if the mass exchange is off center. In principle, for an absolutely sharp knife edge, the only deviation from a z translation will be due to variation in the radius of the wheel, which will translate the point of tangency from which the band rolls off the wheel.

The diameter of the NIST-4 wheel was turned on a high-precision diamond turning lathe. The roundness was measured by the NIST

dimensional metrology group, which found that the diameter varies by less than 950 nm around the circumference, so that parasitic translations of the coil center due to wheel run out should not exceed 475 nm. The mechanism can achieve an 80 mm translation that takes 40 seconds. Assuming the diameter varies continuously from its smallest value to largest over 180 degrees of rotation implies a horizontal linear translation of about 40 nm over the full range of motion, which is about 15 degrees. This would suggest an error velocity of about 1 nm/s, which is still a relative error with respect to vertical velocity of 5.0×10^{-7} . Realistically, the knife edge has a radius and will likely be eccentric from the center of the wheel. In practice, error motions during translation appear to be on the order of 1 to 5 micrometers, leading to a relative error motion on the order of a few parts in 10^5 .

The overall results of creating a symmetric field and motion scheme on the NIST-4 watt balance appear promising. Observations taken using the assembled instrument indicate that the product of off-axis mechanical forces and velocities should contribute to the value of the electrical watt at levels below parts in 10^8 . We now must consider the stabilization of the instrument from other perturbing disturbances arising in the environment.

STABILIZATION

Stabilization of a watt balance instrument takes many forms, ranging from the gross temperature, vibration, and electrical isolation of the instrument from its surroundings, including the operation of the experiment in vacuum, to more subtle techniques, such as the arrangement of the balance center of gravity to be coincident with the balance central pivot point, which helps to minimize noise torques on the balance due to low-level accelerations of the balance frame. Suffice to say that NIST-4 is located in an electrically isolated and shielded room, three stories below ground, on a large cement pier, and that the room is temperature controlled to 0.02 °C.

AVERAGING

Despite best efforts to stabilize the environment, vibration along the vertical axis will remain. No structure is infinitely rigid, and relative motion between the magnetic field and coil is inevitable, directly impacting the measurement of velocity. Coil vibrations relative to the magnet of even a few tens of nanometers produce noise at the

level of a few parts in 10^5 . These are not correlated with a constant velocity, so can in principle be filtered through averaging, however, there are practical limits on how long one can average. For example, the determination of a single estimate of BL may take on the order of 40 seconds. If the vibration noise causes variance on the order of a part in 10^6 in the measured BL , then we will need to average over 10000 such measurements so that the deviation of the mean achieves the desired precision of a few parts in 10^8 . Fortunately, there is a final bit of symmetry that can be exploited. Notice that $BL = U/v_z$. If we assume there is vibration noise, δv_z , then

$$BL = \frac{U + \delta U}{v_z + \delta v_z} = \frac{BL(v_z + \delta v_z) + \delta U}{v_z + \delta v_z} = BL + \sim \frac{\delta U}{v_z} \quad (6)$$

where δU represents physically uncorrelated noise in the voltage detection equipment. Equation (6) illustrates that to first approximation the voltage noise due to vibrations of the coil is common mode and drops out of the ratio.

CONCLUSION

At this stage, we will hope that the electrical noise can be kept below a few hundred nanovolts, so that simple averaging over a few hundred samples will do the trick. Of course, there is more to the complete story of a watt balance that includes measuring the velocity with an interferometer, the synchronization of the voltage and velocity measurements to achieve the common mode rejection, and many other instances where the canon of precision must be applied with vigilance and creativity.

ACKNOWLEDGEMENT

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OPTOMECHANICAL MOTION SENSORS

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INTRODUCTION

Low-loss mechanical oscillators are a unique interface to observe the physics governing the dynamic interactions of light with matter and its environment, yielding sensors of unprecedented resolution. The reflection spectrum of an optical cavity is exquisitely sensitive to motion-driven frequency changes which, when combined with mechanical oscillators, can provide outstanding observation resolution and bandwidth.

Furthermore, a coherent low-noise optical source coupled to a mechanical oscillator provides an optimal observable in its wavelength/frequency, which faithfully reflects the motion of a test mass and the underlying phenomena driving it. The displacement-resulting frequency changes of the light are directly traceable to the International System of Units (SI), resulting in the decisive measurement accuracy capabilities coveted in reference standards.

Based on this premise, we have directed efforts to the development of high sensitivity optical micro-cavities for metrology applications. These cavities consist primarily of fiber-optic micro-mirrors that we have merged with low-loss mechanical oscillators. The main goal of these efforts is to build highly sensitive traceable displacement, force, and acceleration sensors.

This article presents our results on novel and highly compact optomechanical devices that offer both high sensitivity and direct SI-traceability, outlining a path for reference measurements. We show our results on low and high finesse Fabry-Pérot fiber micro-cavities, and the achieved measurement performance.

FIBER-CAVITY DISPLACEMENT SENSOR

Our optical detection scheme is based on two fiber-optic facets facing each other^[1, 2] and separated by a short distance, typically between 40–200 μm .

We have further developed these displacement sensors into two types: (a) a low-finesse fiber

cavity consisting of two flat fiber end mirrors, and (b) a high-finesse plano-concave fiber cavity with highly reflective dielectric coatings as depicted in Figure 1(a) and (b), respectively.

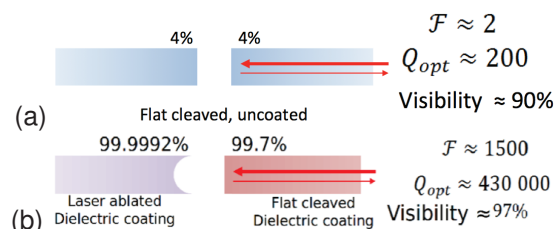


FIGURE 1. Schematics of fiber-optic Fabry-Pérot cavities of (a) low and (b) high finesse.

While the high-finesse cavity certainly provides greater sensitivity to cavity length changes, the dynamic range is significantly lower than the low-finesse counterpart due to its narrow linewidth.

We have been able to achieve displacement sensitivities at levels of $10^{-14} \text{ m}/\sqrt{\text{Hz}}$ with low-finesse fiber sensors. High-finesse fiber sensors have demonstrated displacement sensitivities at levels of $10^{-16} \text{ m}/\sqrt{\text{Hz}}$. Figure 2 shows typical displacement sensitivities that can be measured with these systems in our laboratory.

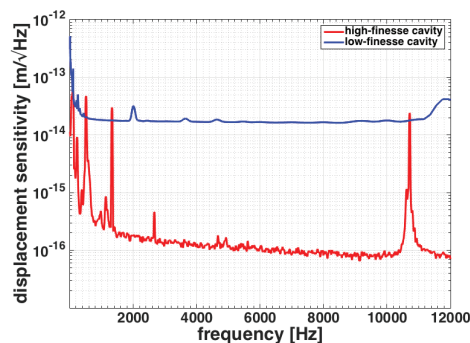


FIGURE 2. Displacement sensitivity measurements with low (blue) and high (red) finesse fiber-optic Fabry-Pérot cavities.

Fiber-based Fabry-Pérot micro-cavity sensors operating in reflection have been incorporated onto fused-silica mechanical oscillators that are equipped with collinear V-grooves as alignment

canals for the fibers, as shown in Figure 3. As a reference on the physical dimensions of these systems, the thickness of the optical fiber is $125\text{ }\mu\text{m}$, and cavity lengths are in order of $100\text{ }\mu\text{m}$. The upper row is a high-finesse cavity with a dielectrically coated injection fiber on the right side and a gold coated concave mirror on the left. The lower row shows a low-finesse cavity consisting of two flat-cleaved uncoated optical fibers.

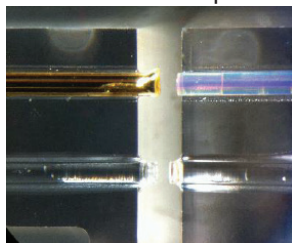


FIGURE 3. Photograph of fiber-optic Fabry-Pérot micro-cavities affixed to the V-grooves of the optomechanical devices.

The fused-silica mechanical oscillators have been designed using slightly different geometries for optimal operation in applications of force or acceleration sensing.

The following sections describe the two types of optomechanical motion sensors that we have developed for such metrology applications and their present performance.

OPTOMECHANICAL FORCE SENSOR WITH FEMTONEWTON RESOLUTION

We have developed an ultrasensitive optomechanical force sensor[3] designed to improve the accuracy and precision of measurements in atomic force microscopy. The sensors reach quality factors of 4.3×10^6 and force resolution on the femtonewton scale at room temperature. Self-calibration of the sensor is accomplished using radiation pressure to create a reference force. Self-calibration enables in situ calibration of the sensor in extreme environments, such as cryogenic ultra-high vacuum. The sensor technology presents a viable route to force measurements at the atomic scale with uncertainties below the percent level.

Figure 4 provides an overview of the self-calibrating optomechanical force sensor.

The sensors are laser machined from a fused silica wafer. The sensor geometry is a parallelogram flexure mechanically grounded at the base with a proof mass at the distal end. It has been

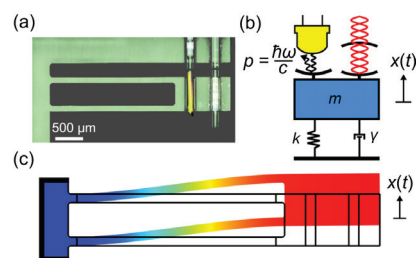


FIGURE 4. Self-calibrating optomechanical sensor. (a) Optical image and (b) schematic of the sensor indicating a high-reflectivity mirror for actuation via radiation pressure and Fabry-Pérot cavity for interferometric displacement measurement. (c) The fundamental flexure eigenmode predicted by a finite element model showing rectilinear displacement of the proof mass.

designed to approximate the behavior of a single-degree-of-freedom (SDOF) oscillator where the transverse displacement of the proof mass in the fundamental eigenmode is approximately rectilinear (see Figure 4(c)), which allows determination of the sensor stiffness through addition of a relatively large mass. In this case, the added mass can easily be as large as $100\text{ }\mu\text{g}$, making it possible to calibrate with a precision microbalance. There are two optical cavities between the proof mass and the support. On the left, low-coherence light from a superluminescent diode is supplied by an injection fiber. The opposing face is a high-reflectivity mirror consisting of a cleaved, gold-coated fiber that actuates sensor oscillations through radiation pressure. On the right is a low-finesse, Fabry-Pérot optical cavity, formed by cleaved, uncoated optical fibers that are used to measure the displacement of the proof mass. The fibers are axially aligned and affixed to integrated v-grooves. The optical power circulating in the cavity is approximately $20\text{ }\mu\text{W}$, therefore the optical spring effect can be neglected. The displacement noise floor of this low finesse fiber cavity is $20\text{ fm}/\sqrt{\text{Hz}}$.

Calibration of the sensor can be accomplished through the direct application of a known reference force, such as radiation pressure from light incident on the test mass. A fiber-optic cavity provides a mechanism for actuation, where the injection fiber is affixed to the base of the sensor, and the opposite fiber mirror is rigidly mounted onto the test mass. Calibration of the force sensor is then accomplished by a ring-down, ring-up sequence. The resonance frequency ω_o and quality factor Q are determined from the ring-

down response. Subsequently, the oscillator is self-excited by the modulating light intensity with a phase-locked loop (PLL), and the sensor stiffness is determined from the steady-state amplitude of the resulting limit cycle.

Figure 5 shows the ring-down test and frequency stability measurement for the sensor at room temperature in high vacuum (10^{-4} Pa). The sensor achieves a quality factor of 4.32×10^6 , which represents a 100-fold improvement over state-of-the-art quartz tuning fork sensors. The high quality factor arises from use of low-loss materials, such as fused-silica, and by careful mounting of the sensor to minimize support loss.

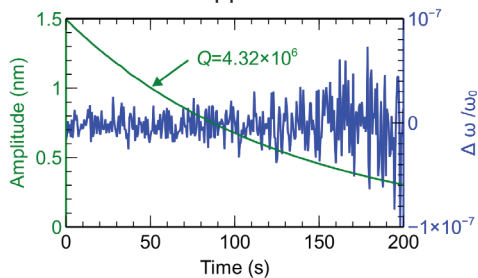


FIGURE 5. Ring-down test showing the amplitude decay and frequency stability the sensor. $Q = 4.32 \times 10^6$ is obtained from a least-squares fit to the amplitude decay. The test was performed at room temperature in high vacuum (10^{-4} Pa)

Assuming the stiffness value from the added mass calibration, we instead use the sensor to measure the radiation force. Thus, the light source is modulated at ω_o by the PLL. The modulation amplitude is then alternated between a pair of discrete, closely spaced RMS intensities. The radiation force is then measured by repeatedly switching the source intensity. From this procedure, we obtained a standard deviation of 67 fN and a force resolution of 14 fN. The resulting distributions are plotted in Figure 6.

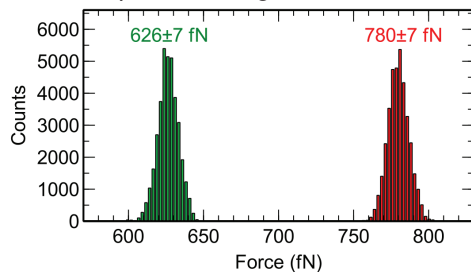


FIGURE 6. Measurement of the radiation force for an alternating source intensity. The standard deviation of the distributions is 67 fN and an estimated force resolution is 14 fN.

HIGH SENSITIVITY REFERENCE OPTOMECHANICAL ACCELEROMETERS

Here, we described our results on optomechanical reference accelerometers that consist of a low-loss fused-silica oscillator, incorporating fiber-optic Fabry-Pérot microcavities to measure the test mass displacement. We have developed acceleration sensors that operate with the two types of fiber cavities described above, low[4] and high[5] finesse.

Assuming a simple harmonic oscillator response for the mechanical resonator, acceleration can be obtained from a direct displacement measurement of the test mass by

$$\frac{X(\omega)}{A(\omega)} = -\frac{1}{\omega_o^2 - \omega^2 + i\frac{\omega_o}{Q}\omega}, \quad (1)$$

where $X(\omega)$ is the relative displacement given an input acceleration $A(\omega)$ at an angular frequency ω , with a harmonic oscillator response of the mechanics determined by its natural frequency ω_o and quality factor Q .

Our devices provide measurement traceability to the SI, since the test mass displacement measurement is directly linked to the laser wavelength. Moreover, the mechanical oscillator translates external accelerations into displacement of its test mass through two parameters only, its natural frequency ω_o and quality factor Q . The parameters can be obtained from series of ring down measurements and spectroscopy analysis that are referred to a frequency standard. The mechanical fused-silica oscillator is shown in Figure 7. This material was chosen for its compatibility with fiber optics and its inherent low loss characteristics.

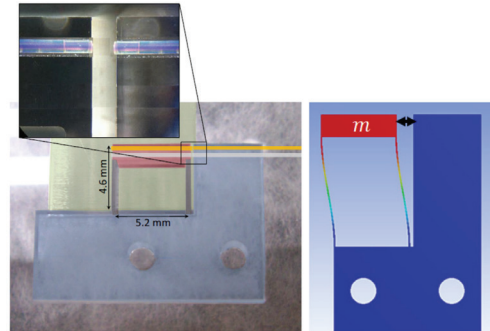


FIGURE 7. Photograph (left) and sketch (right) of the monolithic fused-silica mechanical oscillator with the integrated fiber micro-cavity (magnified in the upper left corner)

We have tested extensively both systems in our laboratory, low and high finesse accelerometers, and have been able to demonstrate acceleration noise floors at levels of $8 \mu\text{g}/\sqrt{\text{Hz}}$ in the low finesse case, and better than $100 \text{ ng}/\sqrt{\text{Hz}}$ over 10 kHz when measuring with high finesse fiber cavities. Figures 8 and 9 show the corresponding measured performances.

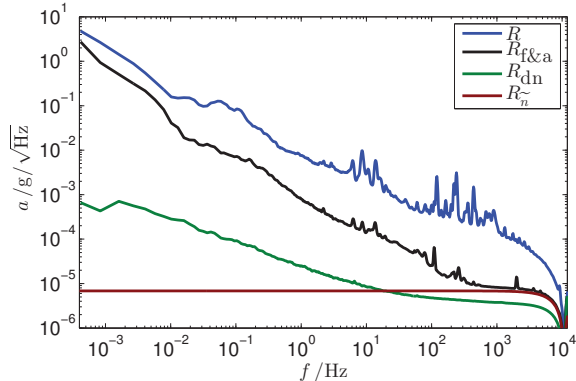


FIGURE 8. Measured acceleration spectra of a low finesse accelerometer with ($R_{f\&a}$) and without stabilizations (R) running. Shown are also the estimated shot noise level (R_n) and the measured dark noise (R_{dn}).

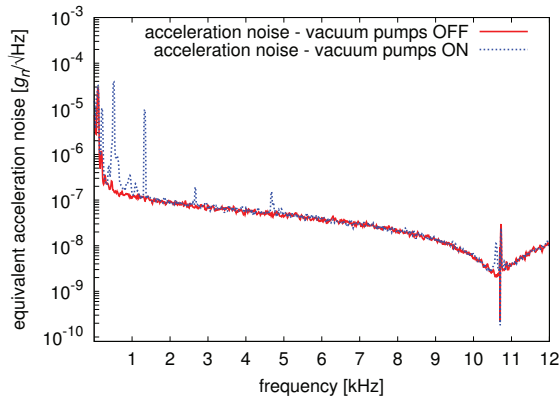


FIGURE 9. Noise equivalent acceleration of a high finesse system, demonstrating sensitivities below $100 \text{ ng}/\sqrt{\text{Hz}}$ over 10 kHz .

A thorough uncertainty analysis of these systems is provided in Reference [4]. Our analysis results estimate that both, low and high finesse systems currently achieve relative accuracies in the order of 10^{-4} . This indicates that our uncertainty levels are better than current state-of-the-art calibration capabilities at National Metrology Institutes[6].

The simplicity, compactness, and traceable high-sensitivity performance over a wide bandwidth accompanied by built-in advanced laser-

interferometric detection highlight our optomechanical sensors for a wide variety of precision measurement applications. The concept of self-calibration and the exquisite performance demonstrated by our devices outline a realistic path towards developments in sensor technology where low uncertainties are crucial.

DISCLAIMER

Certain commercial equipment, instruments, processes or materials are identified in this paper for completeness. Such identification does not imply a recommendation or endorsement by the National Institute of Standards and Technology.

ACKNOWLEDGEMENTS

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CLEARING THE FOG FOR BEST IN THE WORLD AIR-WAVELENGTH

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INTRODUCTION

Laser interferometry is the basis for modern length metrology and achieves very high accuracies as a consequence of the stable, well-known frequencies of laser sources. However, length measurements in air also require corrections based on precise knowledge of the air refractive index. This uncertainty in the refractive index of air sets the fundamental limitation for practical measurements, preventing us from fully utilizing the inherent high accuracy of the laser. Some years ago we developed a refractometer which used a Fabry–Perot (FP) cavity to measure the absolute refractive index of a gas [1]. The device was accurate to 3×10^{-9} ($k = 2$) for dry gas, but showed humidity-related errors of 49×10^{-9} when measuring moist (lab) air. When two cavities were compared side-by-side, one 143 mm in length and the second 329 mm, the error scaled inversely with length, which can be interpreted as a 7 nm shortening in the length of both cavities when exposed to humidity. We believe the change in length was due to water penetrating into the mirror coatings, which caused their apparent positions to change,—even though the coatings were ion-beam sputter (IBS)—and we present evidence which supports this theory. We aim to quantify humidity errors of the FP cavity and provide a correction mechanism so that the FP cavity can achieve good results for moist gas, comparable to what is achieved for dry gas.

To correct this humidity phenomenon—to clear the fog—we have been developing a wavelength-tracker based on a gas-cell integrated into a heterodyne interferometer. This gas-cell is placed side-by-side with the FP cavity to monitor how each device tracks wavelength as humidity is changed (from 0%RH to 45%RH). We aim to correct the humidity error in the FP cavity to better than 3×10^{-9} : so with a 25 cm gas-cell, the heterodyne interferometer must be stable to 0.5 nm over the 10 h duration of the measurement (or fractional length stability $10^{-11} \text{ Hz}^{-1/2}$). A successful correction at this level would allow the FP cavity to realize air-wavelength an order-of-magnitude more accurate than state-of-the-art.

METHODS AND RESULTS

We first investigated the influence of humidity on IBS-coated mirrors. We had an ultralow expansion (ULE) glass [2] substrate IBS-coated, such that a 6 mm diameter central region was coated while the surrounding substrate was uncoated. We placed the substrate inside a sealed enclosure, and used a Fizeau interferometer to measure the change in “height” of the mirror coating as the humidity of air inside the enclosure was changed from dry to moist and back again; desiccant dried the air to less than 10 %RH and saline solution brought it to 70 %RH. The mirror height was defined as the phase difference between the uncoated and coated regions of the substrate, as shown in Fig. 1(a) and (b): the *actual* height of the mirror stack is taller than 10 μm and can only be determined if the fringe order is known; our interest was *change* in mirror height and not actual

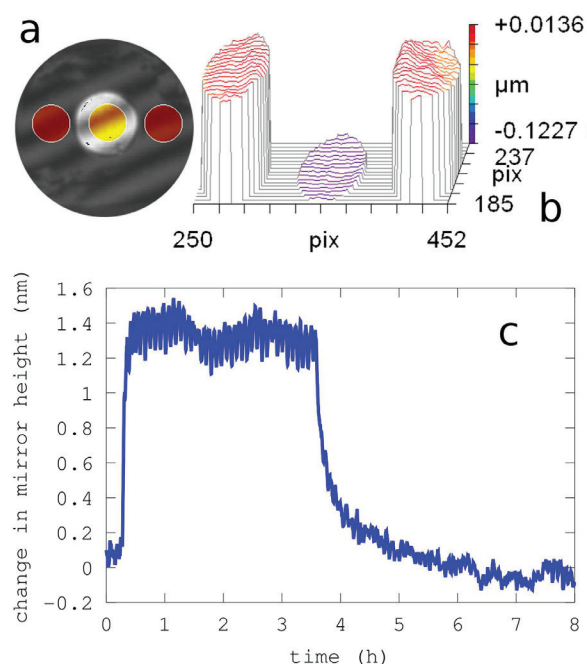


FIGURE 1. (a) Fringes on an IBS-coated mirror with high-reflectivity from the central coated region; (b) mirror height, modulo- 2π with order unknown; (c) change in mirror height as humidity was changed from less than 10 %RH to 70 %RH and back again.

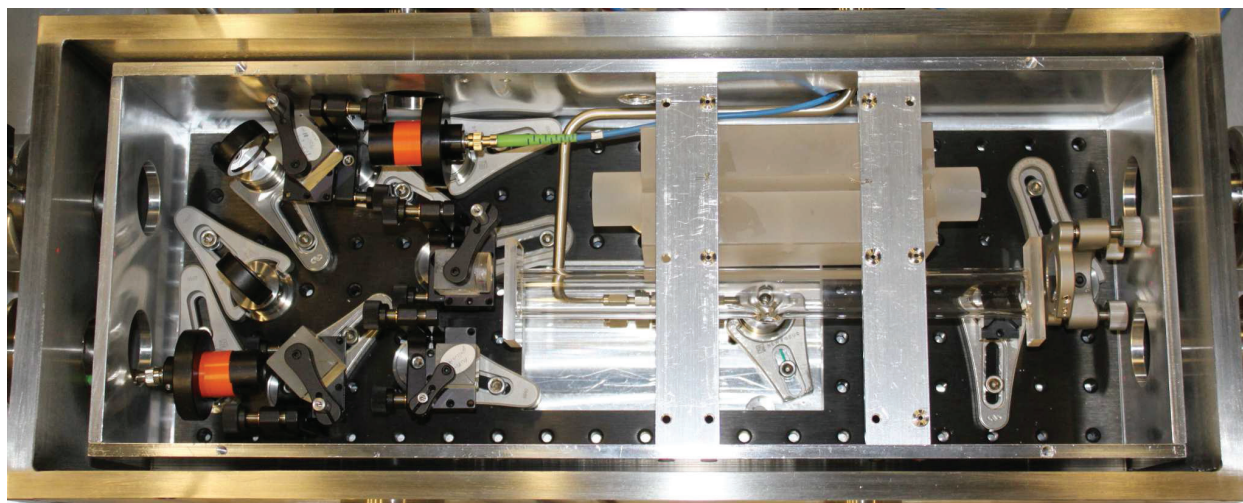
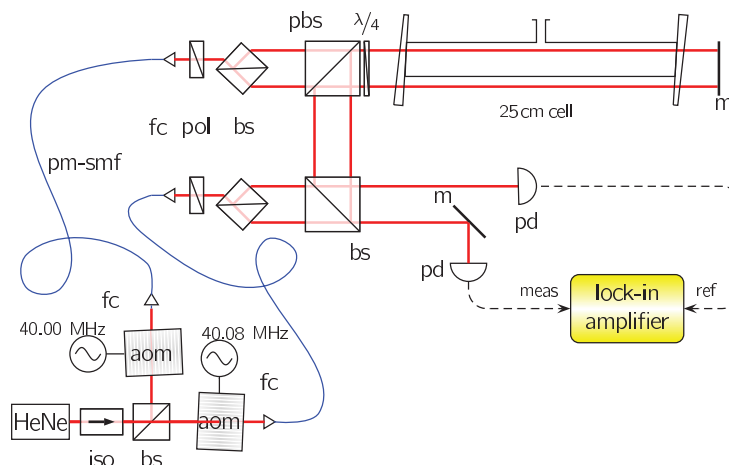


FIGURE 2. Setup of gas-cell heterodyne interferometer. Components include: stabilized helium–neon laser (HeNe), isolator (iso), beamsplitters (bs), acousto-optic modulators (aom), fiber couplers/collimators (fc), polarization-maintaining single-mode fiber (pm-smf), Glan-Taylor polarizers (pol), mirrors (m), and photodetectors (pd). Photograph shows gas-cell and optics beside FP cavity in the temperature-stabilized enclosure.

height. The change in mirror height as humidity was cycled is shown in Fig. 1(c). Several observations can be made from this test: (1) IBS coatings are obviously sensitive to changes in humidity, (2) the apparent mirror position increases in height as humidity is increased, which is consistent with an FP cavity shortening as humidity is increased, (3) the response of the coating to moisture is quite rapid. The one incongruous feature of this test was that Fig. 1(c) shows a change in mirror height of about 1.4 nm; our previous experiments with FP cavities showed a change in mirror height of 3.5 nm (per mirror) when the cavities were brought from vacuum to humid lab air [1]. However, a quantitative comparison of the two results is complicated by the fact that the mir-

ror coatings were not identical in the two experiments.

In order to attempt a humidity correction for the mirrors in our actual FP cavity refractometer we developed a heterodyne interferometer into which a gas-cell was integrated. The setup shown in Fig. 2 is based on the well-known spatially-separated design of Tanaka *et al.* [3], and follows the fiber-fed adaption of Wu *et al.* [4]. The gas-cell interferometer measures the change in path-length between one laser beam passing through the cell interior (reference path) and the beam on its exterior (measurement path). When the cell interior is pumped to high vacuum, the interferometer tracks changes in the refractive index of

air around the cell exterior (such as arise due to changes in humidity). Since the cell windows are uncoated, this interferometer should not suffer from the same water penetration errors that affect our FP cavity; coated surfaces in the interferometer (such as mirrors and beamsplitters) are common to both reference and measurement paths. Also note that correcting the FP cavity only requires the cell to be stable (and precise) as humidity is changed; the errors due to window thickening that usually preclude a cell from measuring absolute refractive index are not a problem.

The gas-cell is made of fused silica and consists of wedged, uncoated windows frit-bonded onto a tube; the tube has a 25 mm outer diameter and is 25 cm long; a short stub teed at the middle of the tube serves as a vacuum port. Off-the-shelf optical components were used throughout, and everything from the output of the fibers was placed on a 10 cm $\times$ 40 cm breadboard. The breadboard was placed beside the FP cavity inside a sealed, temperature-stabilized enclosure, where temperature variations are known to be sub-millikelvin. Our first point of interest was the stability of the interferometer. To test this, we temporarily removed the gas-cell and monitored the stability of the measurement and reference paths in air: the variations in pathlength were less than ± 0.3 nm over 10 h. This stability performance is an order-of-magnitude worse than the phase resolution of the lock-in amplifier, but it nevertheless represents a fractional stability better than 10^{-9} because the cell is configured doublepass. As mentioned above, the goal was to correct the FP cavity to better than 3×10^{-9} .

A more pertinent diagnostic was how well the gas-cell would track changes in the refractive index of lab air when compared to the FP cavity. We know from previous experiments that two FP cavities track one another to better than 1×10^{-9} in normal lab air where humidity is stable to 5 %RH. For this tracking comparison, the gas-cell was placed in the interferometer and its interior was pumped to high vacuum to serve as the reference path. To compare the cell and cavity we relate both to changes in fractional length, which is equivalent to fractional changes in refractive index $\frac{dn}{n} = \frac{dL}{L}$. Length changes in the FP cavity are related to its resonance frequency via $\frac{dL}{L}|_{\text{cavity}} = -\frac{d\nu}{\nu}$. We measure the resonance frequency by beating the cavity resonance against an iodine-stabilized laser and using a counter to

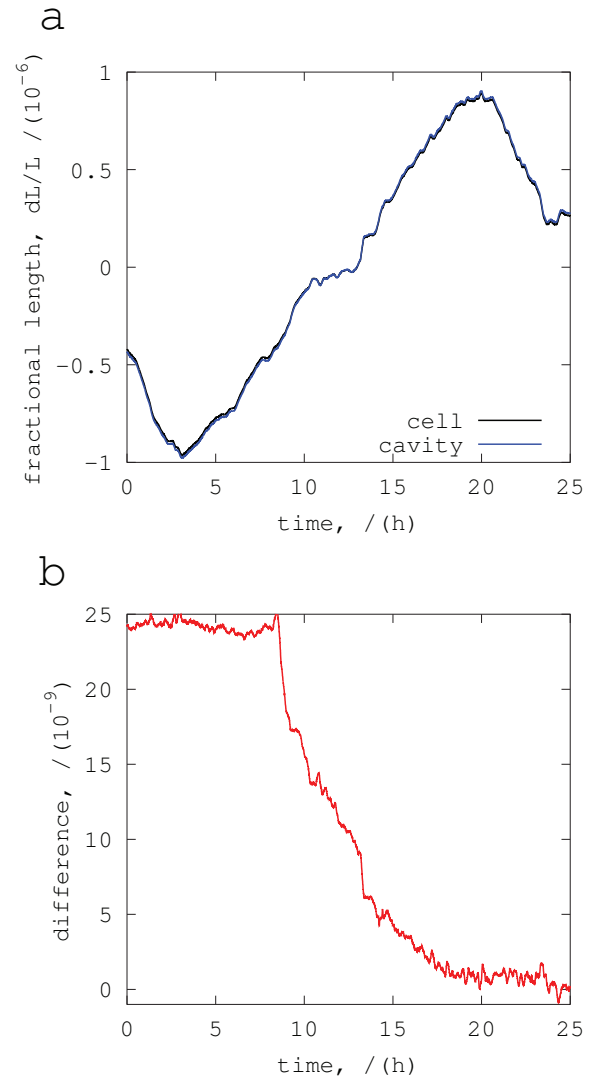


FIGURE 3. (a) Change in fractional length of gas-cell and FP cavity in atmospheric air. (b) The difference between the two: dessicant added at $t=8$ h.

record the beat frequency. Length changes in the gas-cell are related to phase via $\frac{dL}{L}|_{\text{cell}} = \frac{d\phi}{\Phi}$, where $\Phi = \frac{4\pi L}{\lambda}$ and λ is wavelength. We measure the changes in phase between the reference and measurement paths with a lock-in amplifier. We performed a test over 25 h during which the refractive index of air changed by 10^{-6} and observed that the gas-cell tracked the FP cavity to $\pm 3 \times 10^{-9}$. Most of this disagreement in tracking can be attributed to the gas-cell interferometer. Also note, changes of 10^{-6} in the refractive index of air requires knowledge of cell

length to 0.25 mm in order to accurately track 10^{-9} changes. Thus far we only have a caliper measurement of cell length (accurate to about 1 mm), and we plan to soon measure it more accurately with a coordinate-measuring machine.

We attempted to correct the humidity error in the FP cavity, once again using desiccant to dry the air inside the enclosure to less than 10%RH. We observed tracking performance of the gas-cell and FP cavity as shown in Fig. 3. The 2×10^{-6} change in fractional length in Fig. 3(a) is predominantly due to changes in atmospheric pressure (400 Pa changes refractive index by 10^{-6}), and the addition of desiccant at $t = 8$ h barely contributes to the overall change in refractive index. However, the addition of desiccant is clearly visible in the difference in tracking between the cell and cavity plotted in Fig. 3(b). (Also note the aforementioned typical $\pm 3 \times 10^{-9}$ agreement in tracking between the two in normal lab conditions up until $t = 8$ h.) The difference plotted in Fig. 3(b) is $\frac{dL}{L}|_{\text{cell}} - \frac{dL}{L}|_{\text{cavity}}$, so one interpretation of the test is that the cavity gets longer as the mirror coatings are dried out. This relation between mirror position and humidity is consistent with the mirror height test described above. But as with the mirror height test, we are only seeing about half the expected change: we believe the $L = 143$ mm FP cavity shortens by 7 nm ($\frac{dL}{L} = 49 \times 10^{-9}$) when brought from zero humidity to moist air, but our desiccant test only shows 25×10^{-9} disagreement between the FP cavity and gas-cell.

We do not fully understand why our attempts to correct the humidity error in the FP cavity are showing half the expected error. Our best theory at present is that desiccant does not fully remove moisture from the mirror coating. This theory would also account for the fact that Fig. 4 in Ref. [1] does not pass through zero at zero-humidity: in that test the measure of refractive index by two FP cavities disagreed by 2×10^{-8} in moist air, and still showed 1×10^{-8} disagreement when dried with desiccant; whereas when filled from vacuum with dry air the cavities agreed to 10^{-9} . In order to test this theory we are in the process of making the gas-cell interferometer vacuum-compatible: in addition to some design improvements, the next-generation gas-cell interferometer will feature silicate-bonds to form a quasi-monolithic structure that should deliver the highest stability. We will report on this apparatus in the near future.

CONCLUSION

The most accurate realization of air-wavelength is currently limited by a humidity error in our FP cavity refractometer. We have reported some tests that confirm the error arises due to water penetration into the IBS-coated mirrors. We built a heterodyne gas-cell interferometer to correct this error: in normal lab air the gas-cell interferometer tracked the FP cavity to $\pm 3 \times 10^{-9}$ over 25 h. Our attempt to use this gas-cell interferometer to correct the humidity error in the FP cavity has only been semi-successful: the disagreement between cell and cavity is half what is expected when air is dried with desiccant. We believe this shortfall is due to residual moisture in the mirror coatings that can not be removed by desiccant. To test this hypothesis we are in the process of transitioning to a vacuum-compatible gas-cell interferometer that will allow us to compare the cell and cavity from moist air to a perfectly dry state (ie, vacuum).

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ABSOLUTE REFRACTIVE INDEX SENSING AND TRACKING BASED ON VARIABLE LENGTH VACUUM CELL

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INTRODUCTION

Precise knowledge of the absolute refractive index of air is essential for an accurate measurement for most optical interferometers working under atmospheric conditions. For decades, the refractive index of air has been estimated using the empirical equations by continuous measurements of the environmental conditions in the area (temperature, pressure, humidity, etc.), which are limited based on their underlying correction algorithms to approximately several parts in 10^8 [1, 2].

Several other direct measuring approaches have been proposed to achieve a higher uncertainty but still have limitations. Fabry-Perot (FP) cavities [3] are suitable for static measurement of dry gas absolute refractive index and its uncertainty is mainly depending on how well vacuum-to-atmosphere compression effects in the cavities can be corrected. The synthetic pseudo-wavelength (SPW) [4] method avoids gas-filling or pumping processes. However, the measuring range is strongly dependent on the designed lengths of different cavities. Wavelength trackers are also an option but are limited to tracking changings in wavelength due to refractive index, but require another method to establish the absolute refractive index [5]. Refractometers, on the other hand, can measure the absolute refractive index but typically have limited measurement bandwidths, making them unsuitable for tracking applications.

In this proceedings, we present ongoing work by adapting a new optical configuration to the Fujii, *et al.*, refractometer [6], which is based on a variable length vacuum cell built by The National Institute of Standards and Technology (NIST), to accommodate both absolute refractive index sensing and tracking for real-time refractive index compensation, which is synchronized with the interferometry measurements.

WORKING PRINCIPLE

The refractometer contains four sets of revised Wu-type interferometers [7] as shown in Figure 1. The vertical polarized light from a stabilized HeNe laser ($\lambda=633$ nm) is equally split and sent to two acousto-optic modulators (AOMs), which creates two spatially separated beams with a 70 kHz frequency offset. The two beams will go through a 90:10 (R:T) beam splitter and the two transmitted beams will combine again at PD0 to create a reference interference signal.

One reflected beam is vertically polarized and split into four parallel beams using beam splitters and mirrors. Then, the four beams transmit through a polarizing beam splitter and a quarter wave plate towards the variable length vacuum cell. The cell consists of three windows with specific reflective coatings and a metallic bellow to form a length changeable vacuum cavity. Beam 1 goes through the air and reflects

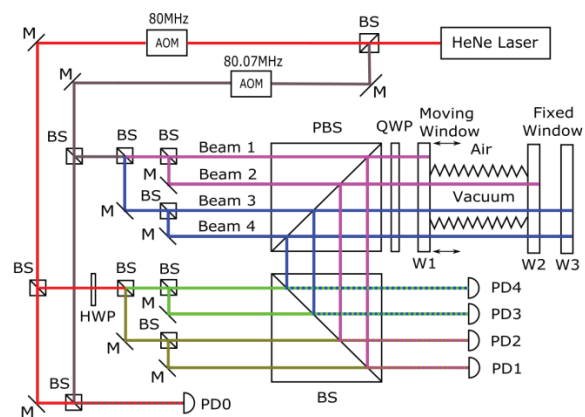


FIGURE 1. Schematic of the refractometer. (BS: beam splitter, M: mirror, AOM: acousto-optic modulator, HWP: half waveplate, QWP: quarter wave plate, PD: photodetector, W: window)

off the first window (moving window). Beam 2 goes through the vacuum cell and reflects off the second window. Beam 3 goes through the vacuum cell and reflects off the third window. Beam 4 goes through air and reflects off the third window. After passing through the quarter wave plate for a second time, the polarization of the four beams rotate 90 degrees and reflect at the PBS.

The second input beam to the system is horizontally polarized after passing through a half wave plate. Then, it splits into four beams which interfere with their respective beams in a 50:50 (R:T) beam splitter. The interference signals can be detected at measurement photodetectors (PD1, PD2, PD3 and PD4) and each optical path length (OPL) change can be determined by comparing the phase difference with reference photodetector PD0.

The refractometer has two working modes: 1) a moving mode to determine the absolute refractive index of air, and 2) a static mode to track its fluctuations. In the moving mode, the moving window (W1) is connected to a linear stage which has a displacement Δz_1 . Due to the force introduced by extension or contraction of metallic bellow, the static windows (W2 and W3) will have a small displacement Δz_2 and Δz_3 . Thus, the optical path length change for each beam will be,

$$OPL_1 = n\Delta z_1 \quad (1)$$

$$OPL_2 = (n-1)\Delta z_1 + \Delta z_2 \quad (2)$$

$$OPL_3 = (n-1)\Delta z_1 - (n-1)\Delta z_2 + n\Delta z_3 \quad (3)$$

$$OPL_4 = n\Delta z_3. \quad (4)$$

The OPL is converted from individual phase change $\Delta\phi$ by

$$OPL = \frac{\Delta\phi}{2\pi} \lambda. \quad (5)$$

The absolute refractive index of air, n , can be determined by solving Equation 1-4,

$$n = 1 + \frac{OPL_3 - OPL_4}{OPL_1 - OPL_2}. \quad (6)$$

In the static mode, the moving window is kept static and refractive index fluctuations can be calculated by tracking the phase change between beam 3 and beam 4 by,

$$\Delta\phi_{3,4} = \Delta\phi_4 - \Delta\phi_3 = \frac{2\pi L}{\lambda} \Delta n + \frac{2\pi(n-1)}{\lambda} \Delta L, \quad (7)$$

where L is the absolute length of vacuum cell and ΔL is the length fluctuation which equals to $\Delta z_1 - \Delta z_2$. In static mode, ΔL is normally equals to

zero. Thus, the air refractive index fluctuation can be determined by,

$$\Delta n = \frac{\lambda}{2\pi L} (\Delta\phi_4 - \Delta\phi_3). \quad (8)$$

By combining the results from these two modes, we are able to achieve absolute refractive index sensing and tracking.

PRELIMINARY RESULT

The experimental process for our refractometer is as follows:

1. Begin with the refractometer at rest and establish a zero point for all interferometers.
2. Scan the refractometer about 5 mm.
3. Establish a starting absolute refractive index value.
4. Hold the refractometer at rest. Monitor the signals to track refractive index change relative to the initial starting value.

The Renishaw XC-80 compensator is placed next to our refractometer which measures air temperature (range: 0°C- 40°C, accuracy: $\pm 0.1^\circ\text{C}$), air pressure (range: 650 - 1150 mbar, accuracy: ± 1 mbar) and relative humidity (range: 0 - 95 %, accuracy: 6 % RH) every seven seconds. All these environmental parameters are input into the NIST Refractive Index of Air Calculator to obtain an absolute value for comparison.

The linear stage is scanning about 5 mm at a speed of 0.25 mm/s over 40 s. The sampling rate for our phasemeter is 200 Hz, which can simultaneously process five signals (four measurement signals and one reference signal). Figure 2 shows the OPL of beam 1 during moving mode, which indicates as equation 1. And figure 3 shows the OPL of beam 2, beam 3 and beam 4 during moving mode, which corresponding to equation 2 to 4. The absolute refractive index is calculated by equation 6.

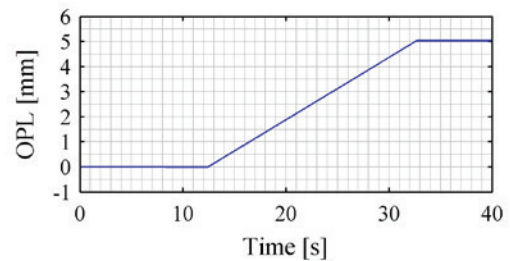


FIGURE 2. The optical path length for beam 1 during moving mode

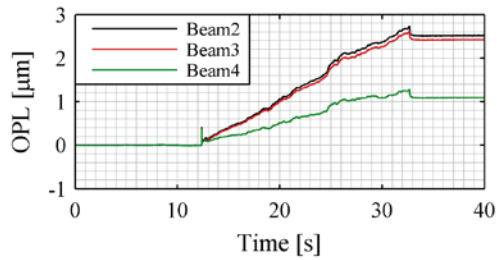


FIGURE 3. The optical path length for beam 2, beam 3 and beam 4 during moving mode

Figure 4 shows the absolute refractive index comparison between our refractometer and Renishaw XC-80. There are totally ten individual tests and the result shows that the mean value for the difference between two setups is 1.3×10^{-8} and the standard deviation is 6.46×10^{-6} .

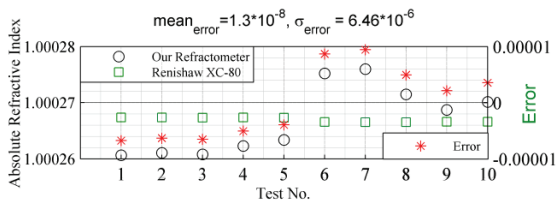


FIGURE 4. The comparison of absolute refractive index between our refractometer and Renishaw XC-80

There are several reasons that might account for the discrepancy. First and most importantly, the linear stage which drives the moving window does not translate smoothly. The signals have some jumps at the start and end position. Secondly, we use a mechanical pump to create the vacuum environment, which is hard to achieve an absolute vacuum. Thus, the refractive index in the vacuum cell is not exactly 1. Lastly, the Renishaw XC-80 compensator has limited accuracy for environmental parameter measurements. Thus, the absolute refractive index calculated by empirical equation has limited uncertainty and can't be treated as true value.

Currently, we only conduct the experiment for absolute refractive index measurement, which is the moving mode for our refractometer. For the static mode which used for tracking the refractive index fluctuation, due to the accuracy for vacuum cell length L and limited bandwidth of Renishaw XC-80 compensator, we are unable

to conduct a comparison experiment at present. We are currently investigating methods to perform this comparison measurement.

CONCLUSION

A new refractometer system that adapting heterodyne interferometers with the variable length vacuum cell formerly used in Fujii refractometer has been proposed, which enables real-time absolute refractive index sensing and tracking. The preliminary experimental result shows the mean value for the difference with Renishaw XC-80 compensator is 1.3×10^{-8} and the standard deviation is 6.46×10^{-6} . Further efforts to improve the measurement accuracy are needed.

FUTURE WORK

We will conduct the comparison experiment that realizes absolute refractive sensing and tracking for a long time period with the environmental sensors. In addition, we plan to replace the PD1 with a quadrant photodetector, which enables the measurement of displacement, pitch and yaw simultaneously for moving window. This information can be used to correct the OPL of different beams and achieve a higher accuracy. At last, better pump and linear stage will be used in our future experiments.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. Jon Pratt and Dr. David Newell for fruitful discussions regarding this work. The authors would like to acknowledge the support of the US Department of Commerce, National Institute of Standards and Technology under Awards No. 70NANB12H186 and No. 70NANB14H266. T. Zhang is grateful for the support of the China Scholarship Council.

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CHALLENGES IN LITHOGRAPHY METROLOGY FROM A ZEISS PERSPECTIVE

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OPTICAL LITHOGRAPHY

The majority of microchips are patterned inside wafer scanners (FIGURE 1) by optical lithography. The pattern on a reticle is usually demagnified by a factor of four and imaged onto the chip wafer with a very precise optical system. ZEISS is the provider of such high end optical systems for all scanners manufactured by ASML, the market leader in wafer scanners.



FIGURE 1. ASML immersion wafer scanner for DUV. ZEISS is OEM supplier of the optical projection systems for all ASML scanners. In the center the projection optics is shown, with the reticle stage at the top and the wafer stage at the bottom.

The last 40 years have seen a drop in the critical dimension (CD) of microchips from 10 μ m down to 10nm. The optical resolution of the projection optics in wafer scanners has to keep up with this development. Since the achievable optical resolution is proportional to the wavelength over numerical aperture, the wavelength was reduced over the years. The first wafer scanners used visible light, but nowadays most high end microchips are patterned with deep ultraviolet (DUV) light at 193nm (FIGURE 2).

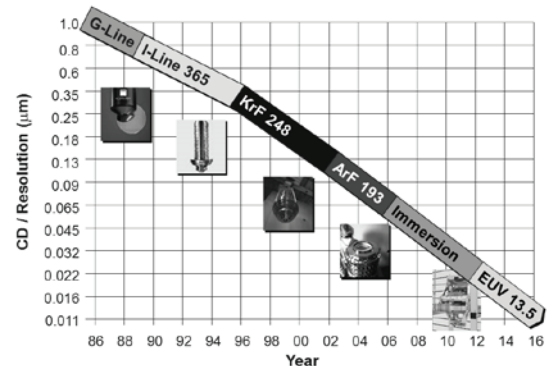


FIGURE 2. The wavelength used in ZEISS projection optics has changed from 436nm down to 13.5nm over the last 30 years.

TRANSITION TO EUV

Feature sizes of less than 35nm require sophisticated multi-patterning techniques with DUV scanners. Therefore more than 15 years ago the development of wafer scanners working with extreme ultraviolet (EUV) light at 13.5nm was started. This reduction in wavelength by a factor of more than 10 raised many challenges in designing and manufacturing the optical system. No material, including air, is transparent at 13.5nm. Some challenges that had to be solved were:

- The optics has to work under vacuum conditions.
- Only mirrors can be used.
- The maximum reflectivity of a surface at normal incidence is ~70%, which can only be achieved with multilayer coatings.
- The reticle has to work in reflection.
- The surface roughness of the mirrors has to be controlled over a wider range
- Optical flare is much more of a problem at 13.5 nm.
- Many optical specifications scale roughly proportional with the wavelength, leading to measurement and production accuracies in the picometer range.

Today the first EUV machines are entering volume manufacturing, proving these optical challenges were all mastered in the end.

THE ROAD AHEAD

Further reducing the CD and supporting the semiconductor roadmap below 10nm will again require an enhancement of the optical resolution while keeping the wavelength unchanged. The way to achieve this, is to increase the numerical aperture from $NA=0.33$ to $NA>0.5$. This change again creates some difficulties:

- Shadowing effects in the reticle coating seem to limit the resolution
- Keeping the light cones separated is geometrically challenging and requires larger incidence angles and incidence angle variations on the mirrors.

This talk will focus on explaining these challenges and those that have been solved to enable EUV scanners. Consequences for the interferometric measurement of EUV mirrors in the production process will be derived and discussed.

EXPERIMENTALLY INVESTIGATE THE ALIGNMENT AND BEAM ABERRATION EFFECTS ON INTERFEROMETRIC DIFFERENTIAL WAVEFRONT SENSING

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INTRODUCTION

Differential wavefront sensing (DWS) has been adopted in several multi degree-of-freedom (DOF) interferometric measuring systems including the New Gravitational wave Observatory (NGO) (formerly known as the Laser Interferometer Space Antenna) [1, 2], which enables simultaneous measurement of displacement, pitch, and yaw. DWS is achieved by analyzing the interference wavefront phase changes on matched pairs on a quadrant photodetector (QPD). The displacement is determined by averaging the phase between the four quadrants. The rotation is determined by creating a weighted phase over symmetrically adjacent quadrant pairs. A fiber coupled three DOF interferometer [3] has been developed targeting industrial metrology areas such as nanopositioning stages, coordinate measuring machines (CMMs), and machine tools, etc. An analytical model and several simulations have been developed to determine the effects of beam diameter, detector size, system alignment errors, and beam wavefront aberrations on DWS signals, which determines the rotational sensitivity [4].

In this proceedings, we compare our fiber coupled 3 DOF interferometer measurement results with a commercial Renishaw displacement interferometer. By measuring the beam diameters, alignment errors, and

wavefront aberrations, we can determine an equivalent length based on the model proposed in [4], which are necessary to precisely estimate the DWS results.

EXPERIMENTAL SETUP

The experimental setup is shown in Figure 1. The left side is our compact, fiber-delivered 3 DOF interferometer which has been previously reported [3]. The right side is a commercial Renishaw angle interferometer. Our system uses a half inch mirror as the target which gives the displacement, pitch and yaw results simultaneously when the stage is moving. The Renishaw system uses a traditional interferometer with a fixed retroreflector pair as the measurement target and is a widely accepted standard for angle calibrations. The two measurement systems calibrate the same motorized linear stage on opposite sides. Since the Renishaw System can only measure one DOF in a single measurement, we chose yaw for comparison. In experiments, the linear stage moves 15 mm in 60 seconds and the two systems measure the same movement simultaneously from two sides at a 100 Hz sampling frequency to our host PC.

Previous simulation results [4] show that beams with a larger diameter incident on a smaller QPD will have a larger linear range and a smaller scaling factor. Specifically, for a perfect

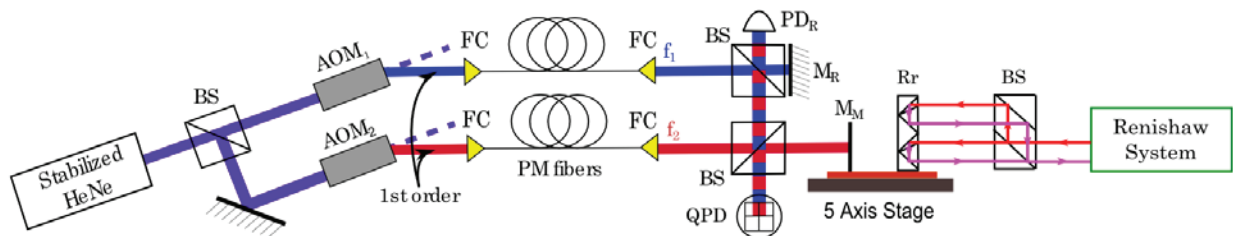


FIGURE 1. Schematic of the experimental setup. (BS: beam splitter, AOM: acousto-optic modulator, PM-SMF: polarization-maintaining single mode fiber, FC: fiber coupler, PD: photodetector, M: mirror, QPD: quadrant photodetector, Rr: retroreflector)

Gaussian beam with 4 mm diameter and detector size is 5 mm by 5 mm, the equivalent length is 3.1384 mm. For the alignment errors simulations, the results show 12.5% beam size mismatch will cause 10% change in sensitivity and 0.5 mm in beam center mismatch will cause 0.7% change in sensitivity. For the beam aberrations simulations, the results show that the power, astigmatism, spherical and 2nd astigmatism will contribute a large change in the equivalent length. Thus, all these system parameters should be measured and considered for a precise rotational sensitivity estimation.

The specification of the collimator shows the collimated beam diameter is 3.4 mm and the detector size is 5 mm by 5 mm. In our initial experiment, we assume the beam is a perfect Gaussian beam without any aberrations and the system is in perfect alignment. The simulated equivalent length based on this assumption is 2.6362 mm. Our yaw measurement result is processed based on this equivalent length and the comparison with the Renishaw measurement result is shown in Figure 2. In this situation, our yaw measurement results do not match the Renishaw results. Thus, the simulated equivalent length based only on nominal beam diameter and detector size is not accurate enough in practice.

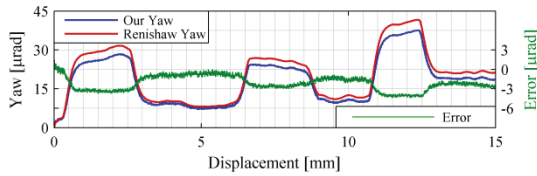


FIGURE 2. The comparison of yaw measurement during linear motion within 15 mm range processed using a simulated effective length 2.6362 mm based on nominal beam diameter and detector size.

To better predict the equivalent length, a Shack-Hartmann wavefront sensor with a $\lambda/50$ rms wavefront sensitivity was used to measure the beam parameters and aberrations. Figure 3 shows the beam intensity profile captured by the sensor, the beam diameter is a weighted diameter calculated as,

$$\frac{2}{\omega^2} = \frac{1}{\omega_x^2} + \frac{1}{\omega_y^2}, \quad (1)$$

where ω_x is the beam diameter in the X-direction and ω_y is the beam diameter in the Y-direction. Thus, the reference beam diameter is 3.24 mm

and the measurement beam diameter is 3.2 mm. In experiments, we tried to balance the power of the four quadrants to ensure the QPD is placed at the interference beam center, which is the coordinate system origin. The reference beam center is (-0.013 mm, 0.016 mm) and the measurement beam center is (0.005 mm, -0.01 mm). The measured coefficients for the first 15 Zernike polynomials are shown in Table 1 based on a 5.411 mm diameter pupil. The simulated equivalent length based on all these measured beam parameters and aberrations is 2.383 mm. Using the aberrated equivalent length, the results show high agreement between both systems (Figure 4).

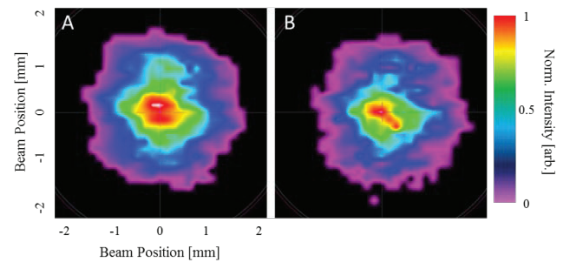


FIGURE 3. Beam intensity profile measurements using a Shack-Hartmann wavefront sensor for the (a) reference beam and (b) measurement beam.

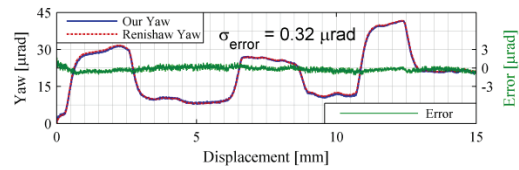


FIGURE 4. The comparison of yaw measurement during linear motion within 15 mm range processed using a simulated effective length 2.383 mm based on measured beam diameter, alignment errors and aberrations.

There are several factors that contribute to the error. First, the beam profile is not exactly a Gaussian beam as we assumed in the simulations. Secondly, there is a refractive index difference between two sides and the sensitivity to refractive index error is different for the two measurement systems. Finally, the measured dynamics for our 12 mm mirror target are slightly different compared to the 70 mm by 30 mm by 30 mm retroreflector target used by the Renishaw system, which results in slightly different dynamic responses when the stage is moving.

TABLE 1. The coefficient for the first 15 Zernike polynomials for reference and measurement beams (unit: micrometers).

Reference Beam	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8
	-0.706	-0.599	0.350	0.373	-0.383	-0.284	0.499
	Z_9	Z_{10}	Z_{11}	Z_{12}	Z_{13}	Z_{14}	Z_{15}
	0.443	0.277	0.268	0.081	0.217	-0.041	0.005
Measurement Beam	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8
	-0.606	-0.681	-0.152	-0.066	-0.153	-0.099	0.419
	Z_9	Z_{10}	Z_{11}	Z_{12}	Z_{13}	Z_{14}	Z_{15}
	0.477	0.207	0.148	-0.028	0.010	-0.018	-0.005

The noise floor over 80 seconds is shown in Figure 5 at a 100 Hz sampling frequency. A constant linear drift of approximately 8 nm was evident when the stage held at a constant position. Thus, the data shown in Figure 5 shows the detrended Z-displacement noise floor. The variations in angle are assumed largely from spatial refractive index variations. This measurement resolution can be further increased through calibration in a better controlled measurement environment.

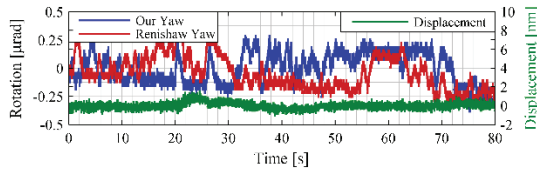


FIGURE 5. The noise floor measurements for pitch, yaw and displacement over 80 seconds for our three DOF interferometer.

After determining the equivalent length, the three DOF interferometer can then be used to calibrate the stage. Figure 6 shows the pitch and yaw measurement result for linear stage moving from 0 to 15 mm, which demonstrates that our interferometer can measure displacement, pitch and yaw simultaneously using a single beam and a small mirror target.

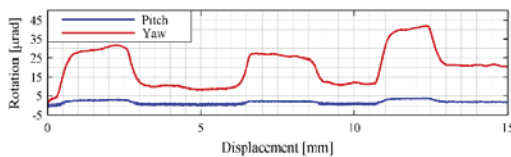


FIGURE 6. Linear stage calibration result within a 15 mm moving range.

CONCLUSION

A fiber-coupled three DOF interferometer has been demonstrated calibrating a linear stage for displacement, pitch, and yaw simultaneously and with high agreement compared to a commercial interferometer. The measurements for beam parameters, alignment errors, and aberrations are necessary to precisely predict the equivalent length in rotational measurement.

FUTURE WORK

Different beam diameters will be adopted and various alignment errors or wavefront aberrations will be created to experimentally investigate how these individual factors affect the DWS results.

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MINIMIZATION OF VIBRATION INDUCED ERRORS USING A GEOMETRICAL APPROACH TO PHASE SHIFTING INTERFEROMETRY

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INTRODUCTION

We are assuming a phase shifting interferometer (PSI) with three or more interferograms collected sequentially using a matrix detector of the size (M, N) . The data from the interferograms are combined together in order to retrieve the phase value $\Phi(m, n)$ at each pixel (m, n) of the detector. We will assume a standard equation describing intensity distribution in the interferograms:

$$A(m, n) + D(m, n) \cos\left(\frac{2\pi}{\lambda} \Phi(m, n) + \delta_i\right) = I_i(m, n) \quad (1)$$

where $A(m, n)$ describes background intensity; $D(m, n)$ corresponds to the modulation parameter $0 \leq D \leq A$ and λ is the wavelength of light; δ_i is a phase step (phase offset) introduced between two consecutive interferograms (i -th and $i+1$) by means of changing the delay between two interfering wavefronts; $I_i(m, n)$ is a measured intensity at location (m, n) in the detector plane; i is an index and defines i -th measurement in the series of p measurements ($i = 1, 2, 3 \dots p$). In this formulation parameters $A(m, n)$, $D(m, n)$ and $\Phi(m, n)$ are three unknowns that need to be calculated, however only phase $\Phi(m, n)$ is of importance in the PSI measurements. As a practical aspect of the measurement, an assumption has to be made that the values of the background intensity and modulation stay constant for the time of data collection.

In order to calculate value of the phase $\Phi(m, n)$, at least three equations with different phase steps δ_i are necessary to solve the system of equations with three unknowns. If the PSI measurement contains only three interferograms the above system is uniquely determined. In case of $i > 3$, the system is overdetermined which allows us to use the existing redundancy to reduce errors in phase calculations. For a good overview of many PSI al-

gorithms and their properties see for example [1].

The solution of the system of equations (1) with $p = 3$ is trivial, however it has a very convenient geometrical interpretation. Let us briefly discuss this interpretation as it is helpful in deriving methods of minimizing errors in phase calculations.

GEOMETRICAL INTERPRETATION OF THE PHASE SHIFTING METHOD

By introducing the following substitutions in equation (1) we can transform the initial problem into a basic set of linear equations:

$$x_1 = A \quad (2)$$

$$x_2 = D \cos\left(\frac{2\pi}{\lambda} \Phi\right) \quad (3)$$

$$x_3 = D \sin\left(\frac{2\pi}{\lambda} \Phi\right) \quad (4)$$

For convenience, in the above notation we have dropped indexes (m, n) but it should be understood that all presented calculations are performed for the same position (m, n) in the interferogram. The above substitutions lead to the following system of equations:

$$\bar{x} \cdot \bar{c}_i = I_i, \quad i = 1, 2, 3 \quad (5)$$

$$\text{where : } \bar{x} = [x_1, x_2, x_3] \text{ and } \bar{c}_i = [1, \cos(\delta_i), -\sin(\delta_i)],$$

which is denoted here as a scalar product of two vectors: the vector of coefficients $\bar{c}_i$ and the vector of unknowns $\bar{x}$. The vector of coefficients represents a vector of constant length (equal to $\sqrt{2}$) which is tilted by a 45° angle with respect

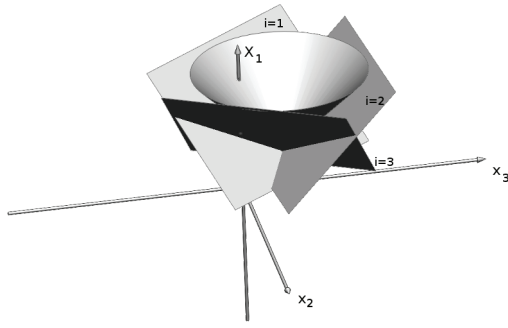


FIGURE 1. Geometrical representation of the system of equations (1) with three interferograms, $p = 3$.

to the x_1 axis and rotated around it by an angle of $(-\delta_i)$. As a consequence any equation in the system of equations (5) represents an equation of a plane normal to the vector of coefficients, i.e. each such plane is also tilted with respect to the x_1 axis by an angle of 45° . Thus, the solution to the system of Equations (5) is represented by an intersection point of three planes in three-dimensional space, each of which is tilted with respect to the x_1 axis by 45° and rotated by an angle $-\delta_i$ around it. The scope of solutions is confined by restrictions pertaining to the measuring system, i.e. by the requirement that the measured intensity $0 \leq I_i \leq I_{max}$.

The equivalent representation of the planes corresponding to the equations in the system (5) can be constructed as a collection of planes being tangent to a cone with an opening angle of 90° , its axis of symmetry along the x_1 axis and apex located at the point representing the solution of the system of equations. Figure 1 shows an illustration of the system of equations (5) with three interferograms, $p = 3$.

If point $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ is the solution of the system of equations (1), then thanks to (2)-(4) we have:

$$D = \sqrt{\hat{x}_2^2 + \hat{x}_3^2}, \quad (6)$$

where D is proportional to the modulation parameter and

$$\Phi = \arctan \frac{\hat{x}_3}{\hat{x}_2} \quad (7)$$

Thus, the modulation can be interpreted as a dis-

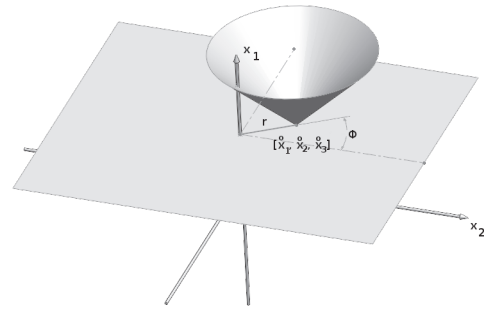


FIGURE 2. Geometrical interpretation of the modulation parameter $D = r$ and the phase value Φ on the plane (x_2, x_3) .

tance from the origin of the coordinate system on the (x_2, x_3) plane and the phase is the angle that the line connecting point $(\hat{x}_2, \hat{x}_3)$ with the origin forms with the axis x_2 . Figure 2 illustrates these relationships.

MEASUREMENTS IN THE PRESENCE OF VIBRATIONS

PSI measurements taken in the conditions of the real world suffer from a variety of problems, most of which cause the phase shifts δ_i to differ from their nominal values. It has been widely accepted that by increasing the number of interferograms it is possible to compensate for their influence on calculated phase. Numerous algorithms were proposed utilizing a larger number of interferograms that allow to minimize specific - typically deterministic - shift errors [2] (these include nonlinearities in the phase shifters or certain modes of vibrations). The geometrical approach presented here allows for a simple interpretation of many strategies applied in those algorithms and offers new options for their development.

It has been commonly recognized that mechanical vibrations are one of the main problems affecting PSI measurements. They produce random distributions of errors in phase steps which in real measurement environments may easily exceed 30° . We would like to present here a new, self-calibrating algorithm [3] aimed at reducing the phase measurement errors caused by the random vibrations.

Assuming a measurement in which three inter-

ferograms are acquired, the three corresponding planes will intersect at one point which typically will not provide any information about the measurement errors. By increasing the number of interferograms, the larger number of intersecting planes will produce a cloud of points instead of a single solution. In this case any subset of three interferograms will produce a single point in the (x_1, x_2, x_3) space. Assuming the p interferograms are collected, they will produce $k = (p - 2)(p - 1)p/6$ of solution points. The spread of this cloud could be considered a measure of a quality of the measurement - i.e a cloud of points confined to a tighter spot would indicate a "better" measurement. The simplest method to utilize the statistical information contained in this cloud is to calculate the center of gravity of the cloud and use it as the solution of the measurement.

As a result, the following algorithm can be constructed:

1. Using information about number of interferograms p and nominal phase shifts values δ_i calculate the coordinates of solution points (x_1^j, x_2^j, x_3^j) , $j = 1, 2, \dots, k$ for each subset of three interferograms from the set (k solutions).
2. Calculate the center of gravity for the k solutions points obtained in the previous step.

$$\bar{x}_i = \frac{1}{k} \sum_{j=1}^k x_i^j, \quad i = 1, 2, 3$$

3. Calculate phase according to Eq.(7) using the center of gravity.

The underlying assumption in this algorithm is that any linearly dependent equations appearing in the first step must be excluded (this condition can be tested by verifying that the three planes in the subsets are not parallel to each other).

This algorithm has a key property of being able to handle any number of interferograms with any phase shifts between them. It also reduces the influence of shift errors but does not eliminate them. It also appears that it should be more effective with the larger number of interferograms (however, in our opinion, this is a debatable statement).

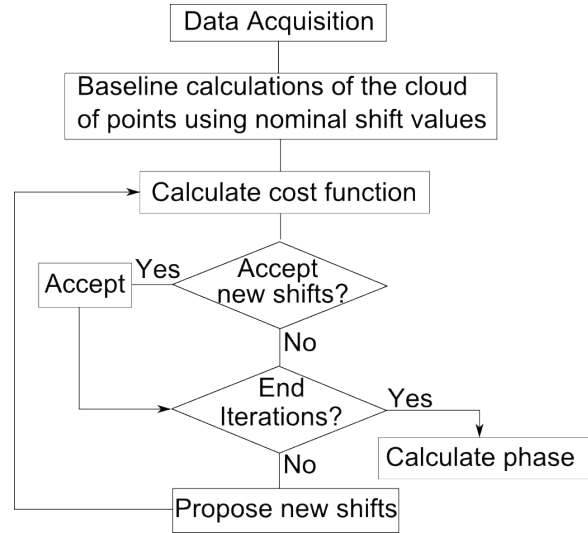


FIGURE 3. Diagram of the basic algorithm.

With this, it is possible to modify the proposed algorithm by complementing the first step with an iterative procedure that would introduce variations into the set of phase steps δ_i . The results of these variations can be evaluated by a specially designed cost function operating on the cloud of points which can use the statistical properties of the cloud. By minimizing the cost function it is possible to arrive at the set of phase steps δ_i which will minimize the extend of the cloud and in turn minimize the errors of the measurement.

The proposed algorithm is very efficient, with execution time only slightly longer then the standard phase calculating algorithms. The iterative search is performed on a random sample of few hundred data points which allows to reduce the execution time and makes it independent from the size of the data array.

EXPERIMENTAL VERIFICATION

The proposed algorithm was tested using a set of simulated measurements in which the interferograms were generated from the phase steps that were modified by random values from within the range of $\pm 60^\circ$. Simulated measurements with 4, 5 and 13 interferograms were tested.

Figure 4 illustrates the results of comparison of the proposed algorithm applied to the 5-frame data set. The upper graph shows the cross section through the phase map calculated by the standard 5 frame algorithm while the lower graph shows the results of the same data processed with the proposed algorithm (please note the dif-

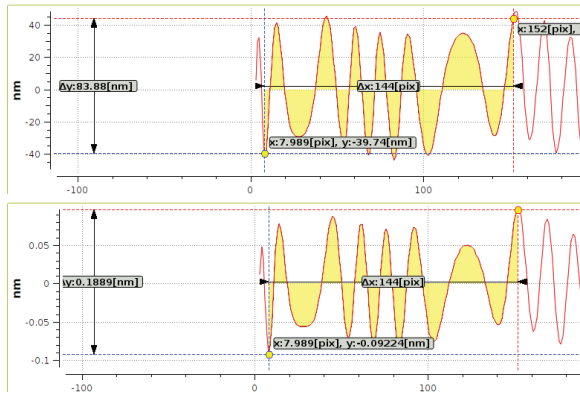


FIGURE 4. Diagram showing a cross section through the phase map calculated from 5 interferograms with nominal phase shifts of 90° . The upper graph shows the result for the standard 5-frame algorithm ($PV = 83.88\text{nm}$) and the lower graph results for the currently proposed, self correcting algorithm ($PV = 0.189\text{nm}$). The corrected steps were calculated as $[0^\circ, 74.8^\circ, 136.4^\circ, 69.3^\circ, 79.6^\circ]$.

ferences in scale). The corrected phase shifts were calculated as $[0^\circ, 74.8^\circ, 136.4^\circ, 69.3^\circ, 79.6^\circ]$.

The proposed algorithm was also applied to the real data. Figure 5 shows the result of a measurement of a spherical surface on a standard Fizeau interferometer with transmission sphere with $F\#/1.0$. Four interferograms were acquired with the nominal phase shift value of 90° between them. The phase map on the left shows the results of phase calculations with a standard 4-frame algorithm and the one on the right is the result of applying the currently proposed algorithm to the same data.

CONCLUSIONS

We have demonstrated that with the proposed algorithm it is possible to reduce errors introduced to the PSI measurements by introducing random variations to the phase shifts. The algorithm is capable of correcting significant shift errors (comparable to the value of a full phase step) and can operate on measurements involving different number of interferograms. Also, it produces better results in cases with fewer interferograms, such as when $p = 4 - 5$. This could be explained by the fact that with fewer interferograms it is easier to search the space of parameters and the algorithm converges faster to the final solution. On the other hand, because of the nature of the proposed approach, the gain from increasing the number of

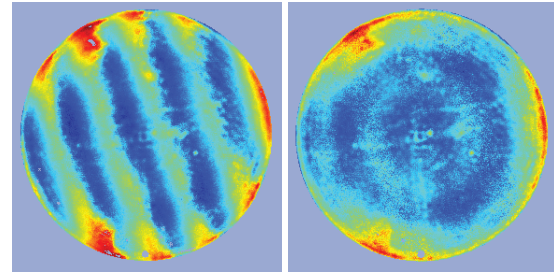


FIGURE 5. Phase map showing calculated phase using a standard 4 frame algorithm with 90° phase shifts on the left and using the current algorithm on the right. Both results were obtained from the same data; the corrected phase steps were calculated as $[0^\circ, 84.1^\circ, 85.3^\circ, 28.9^\circ]$.

interferograms is very limited.

It is worth noticing, that while the algorithm is capable of reducing the effects of errors in phase shifts it can not correct for other problems that may be present in the PSI measurements. In particular, it requires that the modulation D and background intensity A parameters must stay constant during the acquisition time. For example, with long exposure times the image of the interference fringes may get blurred due to vibrations, but this kind of problem can not be corrected with the proposed algorithm.

A somewhat similar approach to the self-calibrating PSI algorithm was recently proposed in the paper by L. Deck [4]

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NON-CONTACT FREEFORM MEASUREMENTS WITH THE MFU200

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ABSTRACT

The MarForm MFU200 is a new high accuracy cylinder coordinate measuring machine which can not only be used for the measurement of high precision aspheric lenses but also for freeform lenses. A critical component is the optical sensor system which was especially developed for this kind of application. Comparing freeforms with rotationally symmetric aspheres additional tasks have to be solved. One is the creation of the measuring process, as a lot of parts do not anymore have a circular outline. Another task is the definition and measurement of a coordinate system for the sample. Finally the analysis of the measured data including fitting procedures has to be solved in new ways. First approaches to fulfil these tasks are discussed.

The performance of the measuring instrument in combination with the optical probe system is demonstrated.

INTRODUCTION / SYSTEM DESCRIPTION

In optical industry freeform surfaces are increasingly applied. This has an influence on all development and production stages, i.e. optical design, description formats, manufacturing, metrology and assembly. A lot of work is going on to address the problems occurring in these processes. A critical step in production is the measurement used for process optimization. So far metrology systems, which are capable to measure freeforms particularly considering flexibility and high accuracy, are hardly available.

The new high accuracy cylinder coordinate measuring machine MarForm MFU200 was recently presented for the measurement of high precision aspheric lenses using a tactile probe system and an interferometric point sensor [1]. It was shown, that the system is not limited to rotationally symmetric samples, i.e. with the combined movement of the rotational and the two linear axes also freeforms can be measured [2].

The formtester MFU200 is a high accuracy cylinder coordinate measuring machine with two linear measuring axes and one rotational measuring axis (Fig.1). The system is a very stable mechanical setup consisting of high precision axes and an internal compensation system (Fig.2). With this compensation system the measurement values are corrected for any nonsystematic movements of the axis, e.g. due to load changes or a nonrecurrent behavior of the guidings. The system was originally designed for applications in mechanical engineering and automotive industries. As the measuring process for lenses is different to these applications the system was optimized considering the new requirements.

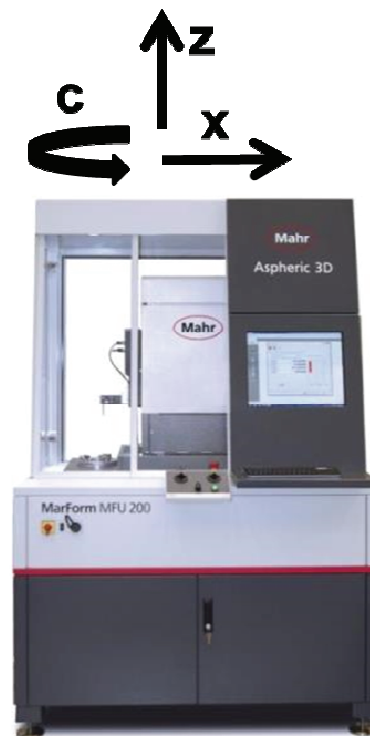


FIGURE 1. Cylinder coordinate measuring instrument MarForm MFU200.

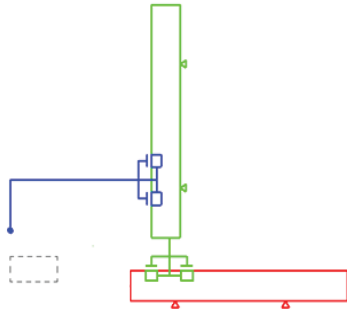


FIGURE 2. Principle of compensation system.

A unique feature is that the system can be equipped with both a tactile and a new type of optical point sensor, thus allowing to directly compare tactile and optical measurements on one system. As optical probe sensor a setup based on white light interferometry has been developed for this special application. The goal was to obtain a high resolution sensor with a real time data processing for tracking processes, i.e. by getting the height values from the probe the system can follow the contour of the sample. In addition the probe system should have a small, lightweight and exchangeable probe, i.e. a small optical system with a fiber connection. The probes can be exchanged via a magnetic contact allowing to switch in between different probes.

As it is desirable to measure optical components without contact the optical sensor is usually favored. However, the optical sensor has other limitations, e.g. the slope of the surface, than the tactile sensor, thus the application of both sensors increases the range of applications. Performing the same measurement task with both sensors allows to compare the performance of the two sensors.

Due to its three high precision and well controlled measuring axis the system is not limited to rotational symmetric samples, i.e. with the combined movement of the rotational and the two linear axes also freeforms can be measured.

OPTICAL SENSOR

A challenge was the development of a small, lightweight probe with a rather high aperture for this application. The problem was not the aperture itself, but an appropriate behavior for this application, i.e. to have a significant signal quality at higher surface angles and angle independent measuring values. Finally angles

slightly above $\pm 20^\circ$ could be achieved. This may not be much, but as it will be used in combination with this metrology setup the probe can be tilted to e.g. 15° , thus an angle up to 35° can be obtained for 3D measurements. With a second or third tilt angle even steeper aspheres can be measured. As aspheres are rotationally symmetric, the probe has to show an angle independent behavior only in one direction. This is demonstrated in Figure 3a) and b). In radial direction the angle of the surface changes continuously whereas for the circular path basically no change of the angle is experienced.

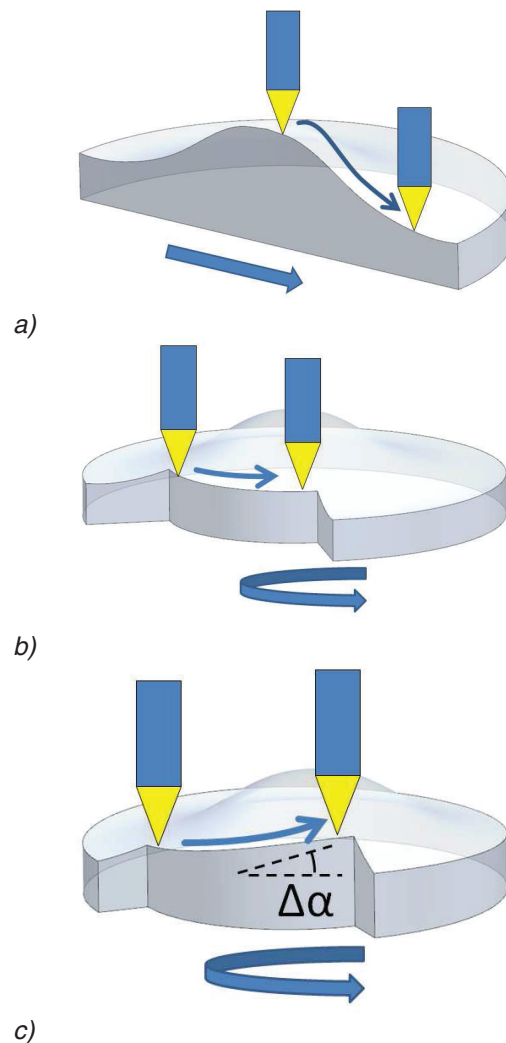


FIGURE 3. a) Linear path of a probe across a rotational symmetric sample. b) Circular path of a probe around a rotational symmetric sample. c) Circular path of a probe around a freeform sample (example off-axis asphere).

If necessary small systematic errors can be compensated in a similar way known from tactile contour systems, where imperfections of the probe tip or ball can be calibrated and compensated. Considering freeforms, i.e. also off-axis aspheres, cylinders and toric shapes, the angle of the surface is also changing on the circular path (see Figure 3c)). Therefore the optical probe has to be angle independent in all directions. Analyzing the performance of the optical sensor system different effects are experienced. In order to be able to deliver a stable system with a sufficient performance the focus of development was the minimization of these effects.

MEASUREMENT OF FREEFORMS

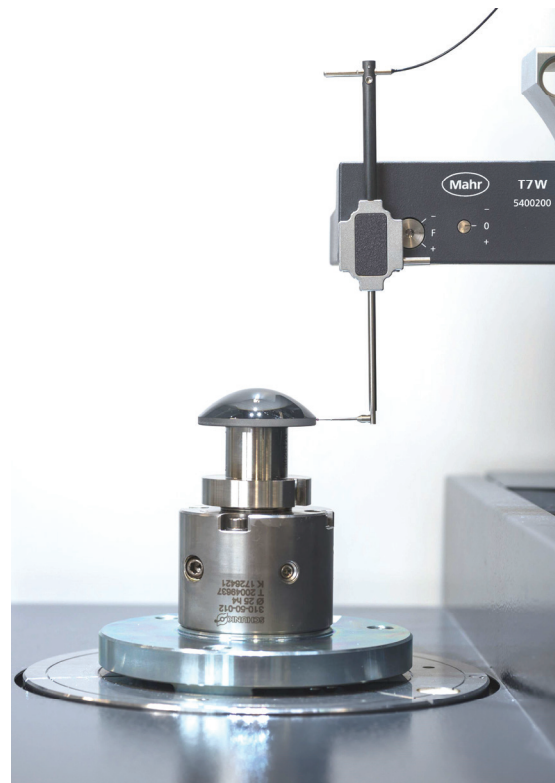
Comparing freeforms with rotationally symmetric aspheres additional tasks have to be solved. One is the creation of the measuring process, as a lot of parts do not anymore have a circular outline. Another task is the definition and measurement of a coordinate system for the sample. Finally the analysis of the measured data including fitting procedures has to be solved in new ways. First approaches to fulfil these tasks are discussed.

For an aspheric lens the creation the measuring process and analyzing the measured data is rather straight forward. Lenses from a grinding and polishing process are usually glued on a holder. Using an appropriate clamping device on the rotational measuring axis (c-axis) the holder guarantees a centered sample without any tilt on that c-axis. Due to its usually circular outer geometry a set of circles and lines can easily be divided over the surface. As the axis of the asphere is very close to the c-axis the measured points can easily be fit into the nominal data of the asphere. For aspheric lenses which are not centered on a holder, they can be centered and, if necessary, tilted. This can be performed with the automatic centering and tilting axis of the MFU200.

For freeforms the situation is much more complicated. For grinding and polishing processes a freeform may also be glued on a holder. However, if the sample is not rotational symmetric the angular orientation has to be found. This may be guaranteed by a special clamping device, which is not a standard, or by a measuring and adjusting process prior to the measuring process.

For e.g. a cylinder lens a circular scan should allow the determination of the orientation. Then,

assuming that the orientation is known, a grid of lines and circles can automatically be distributed over the surface.



a)



b)

FIGURE 4. a) Probe arm with optical and tactile probe tip for the measurement of the freeform surface and reference surfaces.

b) Measureable areas of a sample applying optical and tactile probe tips.

For other freeforms it may be necessary to determine the complete orientation (position and tilt). In this case a simple circular or linear scan

will not be sufficient. But the position of the freeform may be defined by other reference surfaces, e.g. the outer edge of the sample (see Fig. 4a). With the MFU an edge can be measured with a tactile probe in order to determine a coordinate system for the following measuring process with the optical probe and for the analyzing process defining a nominal position. In Fig. 4b) it is demonstrated which part of the sample can be measured. In addition to the edges even the backside can be used as a reference surface.

MEASURING RESULTS

Finally two examples for a measurement of a freeform surface are presented.

In Fig. 5 it is shown the freeform part of a mold of a contact lens. In this case the freeform amplitude is only $\sim 30\mu\text{m}$. For these weak freeform shape the measuring process is rather uncritical: The base shape can be easily centered and the changes of the surface angle during circular scans is very small. Concerning the slope changes during rotation almost no slope changes are experienced. Except for a final fitting of the nominal to measured data the whole process is rather close to a standard process for a rotational symmetric sample.

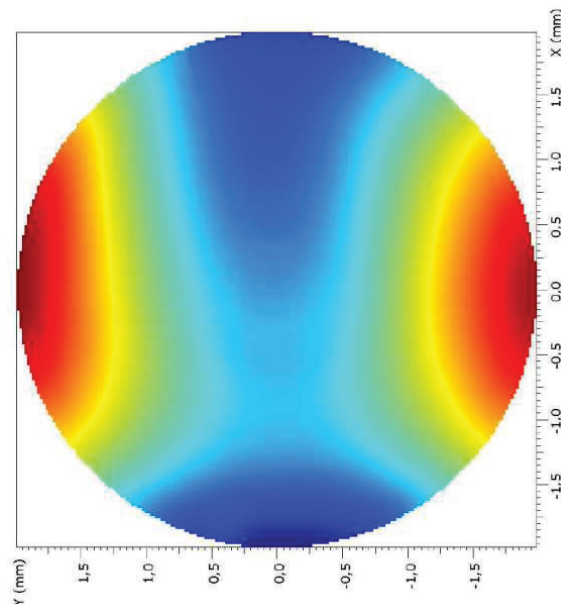


FIGURE 5. Measuring result of a mould of a contact lens after subtracting a best fit sphere.

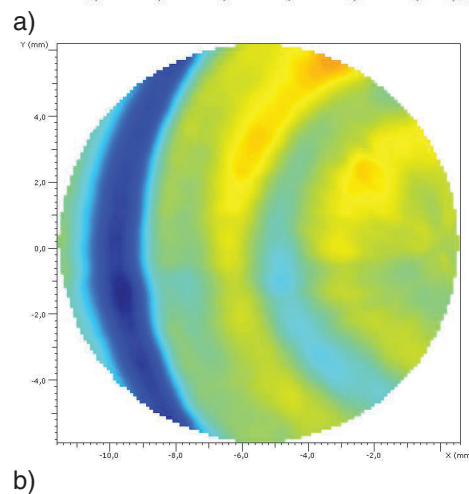
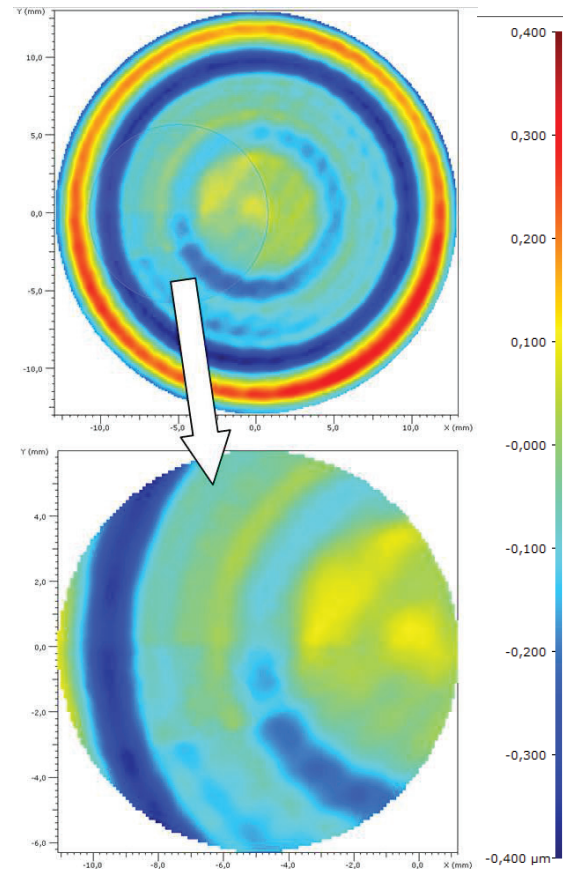


FIGURE 6. a) Measuring result of a rotational symmetric lens and an off-axis area of this result. b) Off-axis measurement (decentered asphere) of this area.

Real freeform surfaces with reference error maps are hardly available. So in the following example a standard aspheric sample is used. It can be measured with a standard process. The measuring result is shown in Figure 6a). It is indicated an off-center area, which, in a second step, is measured in an off-center mode by decentering the asphere on the c-axis of the MFU200. Both results show similar form errors in the range of 500nm. The off-center data are analyzed in an automatic fitting process. As this part of the asphere is slightly tilted compared to the whole asphere, this tilt is also seen comparing both sections.

CONCLUSIONS

Comparing the measurement of rotational symmetric samples such as aspheric lenses with non-rotational symmetric samples (freeforms) different tasks have to be solved. A fundamental component is the optical sensor system. It must be capable to collect measuring data slope independent in all directions even for higher surface slopes. In order to be able to perform measurements and analysis it is important to determine the orientation of the sample. One approach is to determine position and tilt by measuring reference structures. As the MFU200 is capable to use both an optical probe and different kind of tactile probe arms it is very flexible to measure also other structures, e.g. the edges, as references in addition to the optically active freeform surface.

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